TauQuernions $\tau_i,\ \tau_j,\ \tau_k$:

3+1 Dissipative Space out of

Quantum Mechanics

by

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Abstract. We report the discovery of two dual and emergent sets of three *irreversible* bi-vectors - dubbed the *TauQuernions* $\tau_i,\ \tau_j,\ \tau_k$ - that are otherwise isomorphic to quaternions. This inherently dissipative 3-dimensional tauquernion space is a subspace of the geometric (Clifford) algebra $\mathcal{G}_{4,0}$ with generators $\{a, b, c, d\}$; a straightforward mapping produces a 3+1 dimensional sub-space with signature $(+---)$ in $\mathcal{G}_{5,0}$. The individual tauquernions are entanglement operators corresponding to the quantum mechanical Bell and Magic operators. The form $\tau_i + \tau_j + \tau_k$ has 64 sign variants of which 16 are nilpotent, which latter we identify with the Higgs boson; the other 48 variants square to the unitary 4-vector $\pm abcd$, which we identify as the carrier of mass. A natural candidate for dark matter also emerges, which we analyze. We calculate the information content of these and related forms, draw an *exact* map of the entropic pathways an expansion will follow, and sketch how this *Bit Bang* develops. Photons are clearly represented and transparently intertwined in the space, so one can expect overall compatibility with relativity theory.

Keywords: Tauquernions, quaternions, geometric algebra, spacetime algebra, Higgs boson, EPR, entanglement, Bell & Magic operators, space creation, quantum gravity, mass creation, dark matter, black holes, background-independent, string theory, Fourier, Parseval, Clifford, emergence, co-boundary, computational, concurrent, distributed, co-occurrence, co-exclusion, process, mechanism, hierarchical, quantum computing, $q$bit, $e$bit.

1. Introduction

The authors are computer scientists using geometric (Clifford) algebras to describe and investigate the properties of abstract distributed computer systems [11,12,14,15]. In the course of these investigations, we have discovered the quaternion isomorphs, dubbed *tauquernions*, mentioned in the title. We apply this new mathematical description of 3 and $3+1$ dimensions to a contemporary issue: the origin and formation of our $3+1$d universe of 3-space, gravity, mass, time, causality, and entropy, and how all this can emerge from a quantum mechanical soup lacking all of these things. It is important to understand that our results are formally *theory-neutral*, in that they stem from a finite, discrete and combinatorial analysis of the entire phase space.

Our foremost goal here is to describe these novel structures in a straightforward way, so our style is discursive rather than formal. We offer physical interpretations of some of the algebraic forms that appear in order to facilitate the transfer of this structure to the community of physicists. That is, we do computers, not gravity.

1.1 Computational and Physical Processes

One might ask how a mathematics of concurrent computation can come to apply to questions of fundamental physics. There are two pieces to answering this, the first being a mathematics that can connect the two disciplines, and the second, given this mathematics, the details of the connecting isomorphism.

We begin with the common view of a computer program - when it is executing - as a sequence of discrete operations

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where each parenthesis-pair stands for a single such operation. Such a sequence is called a *process,* and the following is a gloss on [11], to which the reader is referred for a more detailed exposition. The process-level of computational description refers not so much to entities themselves as to their interaction, and the sequence of states this produces. In our model, *everything* is a process, or an object built out of processes.

The key property of a process is the exact *order* in which its component operations take place. To capture this ordering property algebraically we will require that each operation "( )" in the above sequence - now viewed as a product - be irreversible (ie. no multiplicative inverse). This prevents algebraic manipulations from changing the effective order.

The algebra we will use is geometric (Clifford) algebra $\mathcal{G}$, a graded vector algebra with both inner and outer products, over the finite field $Z_3 = \{0, 1, -1\}$; Grassmann algebras are a subset of geometric algebra, and the Pauli and Dirac algebras are particular geometric algebras. In fact, geometric algebra is now often advanced as the much-needed common mathematical language for all of physics [3,6,7,10,19]. We have a similar motivation *vis a vis* computer science. The next section introduces geometric algebra; here we anticipate it on a gross level.

A ***Theorem*** of geometric algebra: For any $\mathcal{P} \in \mathcal{G}$, $\mathcal{P}$ is irreversible *iff* $\mathcal{P}$ has an idempotent factor $\hat{X} = \hat{X}^2$.

So make each operation "( )" an idempotent. An idempotent $\hat{X}$ that is also a projector has (in $\mathbb{Z}_3$) the form $\hat{X} = -1 + X$, where X is unitary: $X^2 = 1$. Putting all this together, a sequential process - aka. a measurement sequence - looks like

$$(-1 + X_n)(-1 + X_{n-1})...(-1 + X_1) = \prod_n \hat{X}_i, \qquad X_i^2 = 1$$

This is probably all more or less familiar to physicists. But the *computational* reading of the algebra takes the correspondence much further. In this reading [11], the idempotent form $-1 + X$ is identified as the primitive synchronization operation signal(X), understood to mean "signal the occurrence of the event/state X".

Example. Multiple signallings of the same event's occurrence are semantically equivalent to a single such signal, just as the measurement specified by $\hat{X}$ yields no further information upon being repeated: $\hat{X}^n = \hat{X}$.

Signal's complementary primitive is wait(X), ie. wait for the occurrence (signal) of event X. It is critical to understand that this waiting is not "polling", ie. that the waiting process is constantly and actively checking to see if X has occurred yet, aka. *busy waiting*. Busy-waiting turns out to be a quite untenable view in an asynchronously concurrent universe - something subtler is necessary. A careful analysis [11] reveals that the computational concept of wait(X) must be mapped, speaking now algebraically, to some nilpotent $\omega \in \mathcal{G}$, $\omega^2 = 0$.

In physics, nilpotents supply the causal - and energy conserving - connection between discrete physical events. Wait's play the corresponding role in the synchronizational context - causal connection and conserving information between computational events. Nilpotents are irreversible, so the implication of the above theorem is that we must derive our ω's from our idempotents.

We can derive ω's form by considering two consecutive events $\hat{U}$;$\hat{V}$, forming the process $\hat{V}\hat{U}$. We will insist, now speaking computationally, that $\hat{V}$ *never* occur before $\hat{U}$, ie. the actual process must specify that $\hat{V}$ must always wait (ω) for $\hat{U}$. That is, we want $\hat{V}\hat{U} = \hat{V}\omega\hat{U}$. Rewriting the lhs as $\hat{V}\hat{U} = \hat{V}\hat{V}\hat{U}$ and expanding,

$$(-1+V)(-1+U) = (-1+V)(-1+V)(-U)(-1+U)$$

we find that $\hat{V}\hat{U} = \hat{V}(U - VU)\hat{U}$, and indeed $\omega = U + UV$ is nilpotent so long as U and V anti-commute.[1] Computationally speaking, anti-commutativity means "independent of each other", as in the practice of orthogonal software design, which focuses on ensuring that changes to one module do not affect another; or as in "asynchronously concurrent"; or both, as here.

Processes like $\hat{V}\hat{U}$ are exactly the processes covered by Turing's model of computation, and since entities like $\hat{U}$,$\hat{V}$ are the *projectors* of U, V respectively (so-called *measurement operators*), they are also the (observational) bedrock of quantum mechanics. The key property of such processes - irreversible sequentiality - makes them purely *time-like* processes. It is ultimately this time-like property that allows Penrose to conclude [18] that computational processes cannot capture all the phenomena that quantum mechanics has to offer, among which is entanglement, which is fundamentally space-like.

This prompts the question, "Where then is *space-like* computation"? Which prompts the question, "*What* is space-like computation, what might it *do*?!". An answer to the latter would be, Expand the semantic reach of the computational metaphor to *directly* capture and express fundamental spatial distinctions like *left/right* and *inside/outside*. Given computation as currently practiced, we are forced to simulate (ie. *fake*) such matters, eg. via syntactically sugared high-level languages resting on intricate, and usually sequential, run-time environments.

[The issue is analogous to the background-in/dependence of a physical theory, where string theory *assumes* a $3+1$d background, whereas quantum gravity theories, requiring that 3+1d be *constructed*, are background *in*dependent. In these terms, we are presenting, here, a background-independent, non-supersymmetric, quantum gravity theory.]

Space-like computation, whatever it is, must (for our purposes) provide the equivalent of the quantum potential, with its wave-like properties. Now it is characteristic of a primitive wave that *two* things change at the same time. In the scalar world, these two things could be the x and y coordinates as one traverses the circumference of the unit circle. Computationally, this dynamic corresponds to viewing x and y as independent, but nevertheless jointly interacting, *concurrent* processes that together achieve the required symmetry. [2] That

[1] Note that we could instead write $\hat{V}\hat{U} = \hat{V}\hat{U}\hat{U}$, which leads to $\omega = -V - UV$. This corresponds to the so-called advanced solution, and $\hat{V}\hat{V}\hat{U}$ to the retarded solution.

[2] For example, alternating expression (or possession) of a conserved resource.

is, *rotation* is an example of a space-like *reversible* computation, and is *also* a process.

We accomplish the translation from asynchronous concurrent computational processes to algebraic expressions in the following way. First, we interpret our algebra's "+" sign to mean that (eg.) $U + V$ are two asynchronously occurring and executing, independent, computational entities, ie. processes or objects constructed from same. *Multiplication* is action, transformation, process; both measurement and rotation are examples.

Next, we interpret 1-vectors $a, b, c, ...$ as (reversible) processes possessing *a single bit of state*. These 1-bit processes are deterministic since the one state predicts the next, which alternation encodes frequency ν. Since the grade of the vector equals the number of bits of process state that it encodes, the m-vector ab has $2^{m=2} = 4$ internal states, these being

$$\{a + b, -a + b, a - b, -a - b\}$$

Furthermore, these can be paired as $a + b = -(-a - b)$ and $a - b = -(-a + b)$, and these in turn mapped to the orientation of ab (via the standard and diagonal bases) as:

$$\{a - b, -a + b\} \mapsto +ab \quad \text{and} \quad \{a + b, -a - b\} \mapsto -ab$$

This mapping of states, computationally speaking, allows the whole, ab, to maintain a fixed external appearance – its orientation of either $+1$ or -1 – while at the same time its component processes a and b are themselves undergoing their own (1-bit) state changes. If processes a and b have a stable joint behavior, namely oscillations in one of the above two state-pairs, then ab accurately reflects this in a stable orientation.[3] Furthermore, the computations a-b $\leftrightarrow$ -a+b and a+b $\leftrightarrow$ -a-b, like their algebraic co-respondents, are reversible, both being simple inversions. That is, they are both *wave-like*, ... and at that, *exactly* so (p. 11).

Returning to the introductory paragraph, the mathematical language common to the disciplines of physics and computer science is found, as sketched in the preceding, to be geometric algebra. The connecting isomorphisms are

- 1-*vectors* $a, b, c, ...$ with magnitudes ± 1 represent primitive, reversible *processes* with 1 bit of state; and map frequency ν.
- *m-vectors*, $m > 1$, represent internally-concurrent *process-objects* encoding 2^m bits of state, externally exhibiting orientation ± 1.

[3] The ambiguity regarding the "actual" state of ab leads to the uncertainty principle, and connects with the "nub" mentioned in §2, but we do not pursue this further here.

- Signal(U) is defined to be the idempotent $\hat{U} = -1 + U$, a measurement operator on a unitary U; and Wait(U) is then the nilpotent $U + UV$, with the interpretation that (a later) event $\hat{V}$ is causally connected to (an earlier) event $\hat{U}$.

- Time-like/causal/irreversible *processes* are then Wait/Signal sequences $(\mathsf{WS})^*$.

- The wave-like quantum potential $\Psi \subseteq \mathcal{G}$ *equals* the computational $\mathcal{G}$, constituting an untrammelled, non-deterministic, concurrent computation.

Using this algebra, Matzke [10] found that the quantum entanglement Bell and Magic operators have the form $wx \pm yz$. We show in §7.2 that this form cannot be simulated by a time-like process. It is therefore especially important in the following that the reader understand that when *we* write a sum in the algebra, say $U + V$, *we* are seeing two concurrently executing *objects* U and V, not two dead multivectors belonging to some algebra. [Readers liking conceptual origins might want to read §7.2 first.]

Thus, when we catalog the unitary entities in the geometric algebra $\mathcal{G}_3$ and find exactly three families thereof, whose properties encourage their interpretation as neutrino, electron, and proton/neutron; and we also find three quarkish families $x + yz$, with inherent confinement; along with photons $x + y + z$; and mesons = quark plus anti-quark; etc. etc., all of which matches the standard model to a T (cf. Appendix I); on top of which, it being a fact that $\mathcal{G}_3$ is isomorphic to the Pauli algebra, we find it entirely reasonable to conclude that it *is* real physics that is being described. Perhaps then it will not be so surprising that we find that signals associate to *fermions*, and waits to *bosons*.

The Standard Model having exhausted $\mathcal{G}_3$'s semantic carrying capacity, we graduate seamlessly to $\mathcal{G}_4$ and thence to *construct* 3+1 space as $\mathcal{G}_{1,3}$, along with gravity and mass. Here, among many other corresponding physical phenomena, we find corroboration for earlier proposals [23] linking gravity and quantum entanglement.

Finally, citing [2] (and see also [22]):

> "A minimal quantity of heat, proportional to the thermal energy and called the Landauer bound, is necessarily produced when a classical bit of information is deleted. A direct consequence of this logically irreversible transformation is that the entropy of the environment increases by a finite amount. ... we experimentally show the existence of the Landauer bound ..."

Rolf Landauer (1962): "Information is physical." That is, the now empirically demonstrated physicality of information is what ultimately constitutes the connection between physics and computation.

Add to this the conclusion of Masanes et al. [24] that not only can the standard formalism of quantum mechanics be formally derived from four information-oriented axioms, but as well that their solution is unique. "Bits" are *real* and *cannot* be subdivided. *Information* replaces and generalizes *energy* in their (and our) view. At the other end of the conceptual spectrum, Moreva et al [25] conclude from an entanglement-based experiment (as do we from analysis) that *time* is an emergent phenomenon "deriving from correlations", to which we append that time emerges *solely* from entanglement (§3).

The next section (§1.2) introduces geometric algebra. We then define the quaternion isomorphs in the title of this paper (§2), show how they fit into the Dirac algebra (§3) and why their sum should be identified with the Higgs boson (§4), their relationship to the Bell and Magic quantum entanglement operators (§5), the extension to dark matter (§6), an information-theoretic analysis of these results (§7), and an entropy-driven Bit Bang (§8) that generates all of the foregoing structures.

1.2 Geometric Algebra

For readers unfamiliar with geometric algebra: given a set of anti-commuting 1-dimensional unit vectors $\{a, b, c, ...\}$, these vectors generate the combinatorial space $\{\pm 1, \{a, b, c, ...\}, \{ab, ac, ad, ...\}, \{abc, abd, abe, ...\}, ...\}$ all of which m-vector elements are mutually orthogonal.[4] Thus n generators generate a space of 2^n dimensions. The generators are, simultaneously, the primitive reversible 2-state sequential processes at the bottom of the computational construction. Uniqueness is established by the vector name, ie. we use single character lower case alphabetic characters vs. the matrix column bra-ket notation used with Hilbert spaces. Upper-case letters denote arbitrary multi-vectors, eg. $|A+B| = |A|+|B|$; and the inner product obeys $x{\cdot}Y = xY$, eg. $b{\cdot}ab = -b{\cdot}ba = \tilde{a}$ (see [3,6,7,10] for operator-precedence rules).

We use the canonical geometric algebras $\mathcal{G}_{n,0} = \mathcal{G}_n$, but over $\mathbb{Z}_3 = \{0, 1, -1\}$, so $a^2 = +1$, $a + a = -a = \tilde{a}$, etc., which, in replacing "0,1" with "-1,1", maintains a binary feel, but with vastly expanded semantics compared to Boolean logic and automata theory. We interpret $+a$ to mean that whatever a indicates is currently present, and $\tilde{a}$ that it is not; $0a$ denies a's very existence. [5] Few (if any) of our results apply only in $\mathbb{Z}_3$: certain things – *structural* things – are just easier to see without the additional complexities of multiplicities of identicals.

Geometric algebra's product $ab = a \cdot b + a \wedge b = -ba$ is anti-commutative, but otherwise follows the usual associative and distributive laws. Arbitrary multi-vectors A, B usually neither commute nor anti-commute.

[4] See also http://www.euclideanspace.com/maths/algebra/clifford/index.htm .

[5] We stress that zero is *not* a value, and we would never write "$a = 0$". Rather, zero appears as a situational indicator, eg. "$a + \tilde{a} = 0$", meaning "the occurrence of a excludes the occurrence of $\tilde{a}$". Or $CBA = 0$, meaning that the computation $A; B; C$ has terminated.

All of our calculations have been done with a custom $\mathbb{Z}_3$ geometric algebra symbolic calculator, a Python upgrade of the calculator described in [14]. One should not expect to get the same results from a generic Clifford algebra tool without thoughtful tampering. We use Planck units: $\mathsf{G} = \mathsf{c} = \hbar = \mathsf{k} = 1$.

Notation. Due to the extreme symmetry of $\mathcal{G}_n$ over $\mathbb{Z}_3$, one may safely assume that a given expression is valid for all sign variants unless otherwise noted. Nevertheless, we sometimes write generic expressions using $x, y, ... \in \{a, b, c, ...\}$, with $x, y, ...$ taken without duplication, and all sign variants implied unless otherwise noted. For example, the expression $x - xy$ could denote $a - ab, -a - ab, b - ab, c - cd, \ldots$ but not eg. a or ab alone, nor $a + bc$, nor $a + ab$ (because of the explicit minus sign). To minimize clutter, we use forms with minimal minus signs, and in particular often $1 + x$ rather than $-1 - x$ (the latter being indempotent and the square of the former), even though sometimes it's not quite 'correct'; readers who find this bothersome can just multiply by -1. We sometimes distinguish between the elements of the abstract geometric algebra $\mathcal{G}$ and the subset G that is currently instantiated.

We stress that the various algebraic expressions that we will present and discuss are *discrete computational <u>structures</u>*, eg. 'plus' means 'concurrent'. That is, we view $a, b, ab, ...$ as *local, deterministic processes* whose externally visible states oscillate between ± 1. So the state changes expressed by the algebra represent *concrete discrete computations* producing concrete ie. determinate, non-statistical discrete results. But since the "computation" consists of all possible non-exclusionary processes running flat out concurrently, the familiar non-determistic statistical picture of quantum mechanics nevertheless emerges. This computational view replicates Feynman's sum-over-paths interpretation by realizing, concretely, the Bayesian encoding underlying Dirac's $< V|U >$ bra-ket notation (meaning "the probability of V's occurrence given U's").

In summary, the Heraclitean "everything is process" interpretation that we are placing on the algebra is *quite* different from that of standard treatments of geometric algebra [3,6,7,10,19]. The generators $\{a, b, c, ...\}$ are, ultimately, primitive *distinctions*, encoding only '$\pm$' = 'opposite' in 1d. This expands in >1 dimensions $(ab, abc, \ldots)$ to an m-ary *xnor*, ie. a negated *xor*. It would be a complete misunderstanding to understand our $\mathcal{G}$ expressions as *m.l.t.* formulae. ‘’

2. *The TauQuernions*

The quaternions encode 3d space via the multiplication (= rotation) table:

$\times$	$Q_i = ab$	$Q_j = ac$	$Q_k = bc$
Q_i	-1	$-bc$	ac
Q_j	bc	-1	$-ab$
Q_k	$-ac$	ab	-1

=

$\times$	Q_i	Q_j	Q_k
Q_i	-1	$-Q_k$	Q_j
Q_j	Q_k	-1	$-Q_i$
Q_k	$-Q_j$	Q_i	-1

The corresponding tauquernions are $\mathcal{T}_i = ab - cd$, $\mathcal{T}_j = ac + bd$, $\mathcal{T}_k = ad - bc$. [6] Their multiplication table is below left; on the right is the same table, but with the mapping $1 + abcd \mapsto$ "-1". We emphasize that the tauquernion relationships below are independent of the restriction to $\mathbb{Z}_3$.

$\times$	$\mathcal{T}_i = ab - cd$	$\mathcal{T}_j = ac + bd$	$\mathcal{T}_k = ad - bc$
$\mathcal{T}_i$	$1 + abcd$	$-ad + bc$	$ac + bd$
$\mathcal{T}_j$	$ad - bc$	$1 + abcd$	$-ab + cd$
$\mathcal{T}_k$	$-ac - bd$	$ab - cd$	$1 + abcd$

=

$\times$	$\mathcal{T}_i$	$\mathcal{T}_j$	$\mathcal{T}_k$
$\mathcal{T}_i$	"-1"	-$\mathcal{T}_k$	$\mathcal{T}_j$
$\mathcal{T}_j$	$\mathcal{T}_k$	"-1"	-$\mathcal{T}_i$
$\mathcal{T}_k$	-$\mathcal{T}_j$	$\mathcal{T}_i$	"-1"

Like the Q's, the $\mathcal{T}$'s anti-commute, eg. $\mathcal{T}_i\mathcal{T}_j = -\mathcal{T}_j\mathcal{T}_i$; close circularly, eg. $\mathcal{T}_i\mathcal{T}_k = \mathcal{T}_j$; and $-\mathcal{T}_i\mathcal{T}_j\mathcal{T}_k = \mathcal{T}_k\mathcal{T}_j\mathcal{T}_i$.

One can easily see that the two tables to the right, quaternion and tauquernion, are isomorphic. The tauquernions, elements of $\mathcal{G}_4$, recapitulate in four spatial dimensions what the quaternions, elements of $\mathcal{G}_3$, do in three (but with a twist).

We always operate on the left, so $\mathcal{T}_k\mathcal{T}_j\mathcal{T}_i$ read right-to-left is the sequence $\mathcal{T}_i$; $\mathcal{T}_j$; $\mathcal{T}_k$. This "full circle" rotation defines "+1" = ("-1")^2= $(1 + abcd)^2 = -1 - abcd$, which is idempotent.

In fact, since $wx + yz$ has the idempotent factor $-1 \pm wxyz$, then via the aforementioned theorem (§1.1), *all tauquernions* $wx+yz$ *are* ***irreversible***. The details are revealing:

$$
\begin{aligned}
(ab-cd)^4 &= (ab-cd)[(ab-cd)(ab-cd)](ab-cd) \\
&= (ab-cd)[1+abcd](ab-cd) && [\text{“-1”}] \\
&= (ab-cd)(-ab+cd) && \textit{invert} \\
&= -(ab-cd)(ab-cd) \\
&= -(1+abcd) && \textit{ie. - (-1)} \\
&= -1-abcd && \textit{idempotent} \\
&= \text{“}+1\text{”}
\end{aligned}
$$

[6] There is also a conjugate set: $\{\mathcal{T}'_i\ ,\ \mathcal{T}'_j\ ,\ \mathcal{T}'_k\ \} = \{ab + cd,\ ac - bd,\ ad + bc\}$. There are 8 such triples: $\pm\mathcal{T}_i\pm\mathcal{T}_j\pm\mathcal{T}_k$, and similarly for $\mathcal{T}'$. Choosing a particular triple, as here, constitutes an arbitrary choice of coordinate system orientation, cf. the "right hand rule" in 3d.

which identities also justify our identification of $1+abcd =$ "-1" in the table. The interplay between ± 1 and "± 1" is the interplay of reversible change (space-like, Ψ) and irreversible change (time-like, 2^{nd} Law), and constitutes the scalar nub of what tauquernions do: connect a space-like inversion directly to an *exactly* corresponding time-like inversion: $(-1)(ab-cd) = (1+abcd)(ab-cd)$. [7]

Thus, at least in principle, simply by replacing every (namely reversible) quaternion element xy in one's work with (the irreversible) $xy + wz$, one in effect replaces an explicit time coordinate with an implicit one, perhaps allowing for great simplification.

The conjugate tauquernion table below differs only in negating the Q's:

$\times$	$\mathcal{T}'_i = ad+bc$	$\mathcal{T}'_j = ac-bd$	$\mathcal{T}'_k = ab+cd$
$\mathcal{T}'_i$	$1-abcd$	$ad+bc$	$-ac+bd$
$\mathcal{T}'_j$	$-ad-bc$	$1-abcd$	$ab+cd$
$\mathcal{T}'_k$	$ac-bd$	$-ab-cd$	$1-abcd$

$=$

$\times$	$\mathcal{T}'_i$	$\mathcal{T}'_j$	$\mathcal{T}'_k$
$\mathcal{T}'_i$	"-1"	$\mathcal{T}'_k$	$-\mathcal{T}'_j$
$\mathcal{T}'_j$	-$\mathcal{T}'_k$	"-1"	$\mathcal{T}'_i$
$\mathcal{T}'_k$	$\mathcal{T}'_j$	$-\mathcal{T}'_i$	"-1"

It follows that $-\mathcal{T}'_i\mathcal{T}'_j\mathcal{T}'_k = -1 + abcd =$ "$+1$" just as we earlier saw that $-\mathcal{T}_i\mathcal{T}_j\mathcal{T}_k = -1 - abcd =$ "$+1$".

The table below reifies these mappings for ± 1 [a *sqert* is the square root of an idempotent]:

I	$I^2 =$ "$+1$"	I*'s type*	$\mapsto$
$1-abcd$	$-1+abcd$	$sqert'$	"-1"
$1+abcd$	$-1-abcd$	$sqert$	"-1"
$-1+abcd$	$-1+abcd$	$idem'$	"$+1$"
$-1-abcd$	$-1-abcd$	$idem$	"$+1$"

Taking $1-abcd$ as Minus One (first row above) as an example, then as expected the usual multiplication/sign rules hold:

$(-)\times(-)$	$= -1+abcd$	$+$
$(-)\times(+)$	$= +1-abcd$	$-$
$(+)\times(-)$	$= +1-abcd$	$-$
$(+)\times(+)$	$= -1+abcd$	$+$

[7] This is also a nice example of the familial affinities of the algebras $\mathcal{G}_{mod4}$. Note that $(1 \pm abcd)E = -E$ only if E is an eigen-form of $abcd$ (see later).

Actually, this goes further, since it turns out that $1 \pm abcd$ are examples of a "sparse -1" [14]. Make a "truth table" for the expression $abcd$ and represent the result as a vector, yielding $+abcd = [+ - - + - + + - - + + - + - - +]$ and $-abcd = [- + + - + - - + + - - + - + + -]$. The elements of these vectors form a (generally non-orthogonal) basis for their space.

Let zero $\cdot$ represent a state that does not occur; $\tilde{\mathbf{1}} = [- - - - - - - - - - - - - - - -]$, and $\mathbf{1} = [+ + + + + + + + + + + + + + + +]$. Then $+\mathbf{1} + abcd$ is

	[	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	]
+	[	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	]
=	[	-	•	•	-	•	-	-	•	•	-	-	•	-	•	•	-	]

That is, $1 + abcd = [- \cdot \cdot - \cdot - - \cdot \cdot - - \cdot - \cdot \cdot -] = [- - - - - - - -]$ is a *sparse* -1, and $1 - abcd$ is another. These forms, sparse and otherwise, play a key role in the information-theoretic analysis of §7.

Summing up, we conclude that the two dual sets of tauquernions are each exactly isomorphic to the quaternions, the essence of 3d space, except that tauquernion space is inherently dissipative. This obtains because $\mathcal{T}_i, \mathcal{T}_j, \mathcal{T}_k$ are individually irreversible, as is their sum. Particulate motion in this space is thus thermodynamically governed ie. entropic, and this property encourages us to identify such motion with gravitational free-fall. It follows logically that the two conjugate $\mathcal{T}$-forms describe the two polarization states of gravitational waves, not least because of the following extraordinary unifying connection (apparently overlooked, since it appears in none of the obvious references [3,6,7,10,19]).

Theorem. The projection of a function F onto an orthogonal inner-product space is the Fourier decomposition of F. ■ [17]

Since the elements $a, ab, abc, \ldots$ are all mutually orthogonal, whence $\mathcal{G}_n$ has $\mathcal{O}(2^n)$ dimensions, *every* expression in $\mathcal{G}_n$ is therefore implicitly a *wave operator* as well as the structural description of a computational entity: we can *automatically* impute wave-like properties to both entities (eg. the $\mathcal{T}$'s and their sums) and interactions (products) in the tauquernion space. This gives the whole endeavor a thorough-going hierarchical and holographic/distributed feel, and completely redeems de Broglie's initial insight (1923) of the fundamental wave-like nature of reality.

At the same time, tauquernion space *is* 3d spatially, and so inherently supports the propagation of 3-dimensional waves even though it takes four orthogonal distinctions $\{a, b, c, d\}$ in phase space to construct these 3d waves. We now turn to the algebra of this 3d space.

3. Spacetime Algebra

We seek to define the *spacetime algebra* $\mathcal{G}_{1,3} = \{\gamma_0, \gamma_1, \gamma_2, \gamma_3\}$ with signature $(+---)$.

Perhaps naively, we initially considered mapping *abcd* to the time-like dimension via the vector γ_0: the properties of geometric algebras cycle $mod\,4$, so there is a family resemblance between, say, $\mathcal{G}_0$ and $\mathcal{G}_4$, which is our case in point, since $\mathcal{G}_0$ is the scalar dimension, and similarly, *mass* is a scalar quantity. It is only required that γ_0 square to $+1$, as indeed *abcd* does. This ensures that *abcd* qua mass, and its automatically dissipative motion, isn't pushed into the background. Right or wrong, this approach was abandoned when we discovered the density of the physics and mathematics involved, and so instead we here simply establish the standard formulation as well as we can.

The $\mathcal{G}_4$ tauquernion space, arising out of the quantum spinorial soup, is discrete, and so we can associate a new fifth, unchained dimension t on which to tally a sequence of discrete motions in the 3d tauquernion space. Changes in the state of *abcd* map to the 1-vector t.

The resulting $\mathcal{G}_5$ generators are then $\{a, b, c, d, t\}$. Consider now only the subspace defined by the three tauquernions $\mathcal{T}_i, \mathcal{T}_j, \mathcal{T}_k$ and t. The latter squares to $+1$ while the other three square to "-1", so we have a Lorentzian $(+---)$ space, and it is our understanding that the requirements of special relativity are therefore satisfied; the next section pursues the putative connection to general relativity.

Define now within $\mathcal{G}_5$ the mappings

$$t \mapsto \gamma_0 \qquad \mathcal{T}_i \mapsto \gamma_1 \qquad \mathcal{T}_j \mapsto \gamma_2 \qquad \mathcal{T}_k \mapsto \gamma_3$$

where the γ_i are anti-commuting 1-vectors. The γ_i then generate an explicit basis for the spacetime algebra $\mathcal{G}_{1,3}$:

$$1, \ \{\gamma_i\}, \ \{\gamma_i \wedge \gamma_j\}, \ \{\gamma_i \wedge \gamma_j \wedge \gamma_k\}, \ I = \gamma_0 \wedge \gamma_1 \wedge \gamma_2 \wedge \gamma_3$$

"The structure of this algebra tells us practically all one needs to know about (flat) space time and the Lorentz transormation group" [3, p.131]. We refer the interested reader to [19 §24.4-7, 5, 8] for extended discussions of applying geometric algebra to the standard formalisms of QM and GR.

This said, the derivation of the Dirac equation in [19] points out that the key is that the quaternions can be construed as the square of the D'Alembertian wave operator $\Box$. It follows that if/when the tauquernions, being irreversible, are substituted for the quaternions, it might well be possible to eliminate the explicit time coordinate entirely and end up with the algebra $\mathcal{G}_{0,3}$ (over the tauquernions) as a description of spacetime. [A paper currently in draft extends this to include electro-magnetism.]

Howsoever, every set of $\mathcal{T}$'s also satisfies the basic condition for them to *connect* with each other: that the grade of the parts $ab + cd \mapsto 2 + 2 \leq 4$ not exceed the grade of their union, here $abcd \mapsto 4$. In so doing, and taking advantage of $\mathcal{G}$'s being a coordinate-free algebra, the next section shows that an associated coordinate-free, dissipative, discrete Higgs field then automatically appears as the 3+1d $\mathcal{T}$-coordinate system *itself.*

The transition from this discrete field to a continuous field over $\mathbb{R}$ lies beyond our remit, but we note that the entire algebra lies under the umbrella of Parseval's Identity, and by implication, of harmonic analysis, which latter applies very generally. Indeed, this identity is wave-particle duality in a nutshell.

On the other hand, some writers [1] suggest abandoning $\mathbb{R}$ altogether:

"A key assumption of [the contemporary Theory-of-Everything scene] is that it regards the laws of physics as being the bottom line, and assumes that these laws govern a world of point particles or strings (or other exotica) that is a continuum. Another possibility is that the Universe is not at root a great symmetry but a computation. The ultimate laws of Nature may be akin to software running upon the hardware provided by elementary particles and energy. The laws of physics might then be derived from some more basic principles governing computation and logic. This view might have radical consequences for our appreciation of the subtlety of Nature, for it seems to require that the world is at root discontinuous, like a computation. This makes the Universe a much more complicated place. If we count the number of discontinuous changes that can exist, we find that there are infinitely many more of them than there are continuous changes. By regarding the bedrock structure of the Universe as a continuum we may not just be making an approximation but an infinite simplification."

We note that (1) actually, we show that truly concurrent computation (cf. §7.2) *upholds* the symmetries, cf. the isomorphism between eg. $\mathcal{G}_3$ and the Pauli algebra; and (2) the mentioned "hardware" is crude analogy by our standards - we construct it all (§8).

Howsoever, an entity X existing in phase space $\Psi = G$ comes to occupy $3 + 1$d tauquernion space via the projection $\left(\mathcal{T}_i + \mathcal{T}_j + \mathcal{T}_k\right) \cdot X$, which projection also masks the quantum dimensions automatically, replacing them with $\mathcal{T}_i$, $\mathcal{T}_j$, $\mathcal{T}_k$ and, indirectly, t.

4. The Higgs Boson

We identify $\mathcal{H} = \mathcal{T}_i + \mathcal{T}_j + \mathcal{T}_k$, whence $\mathcal{H}^2 = 0$, with the Higgs boson. To see why, we look more closely,

$$\mathcal{H} = (ab - cd) + (ac + bd) + (ad - bc)$$

$$= ab + ac - bc \ + \ (a + b - c)d$$

from which we see that $\mathcal{H}$ - our space constructor - is a combination of a quaternion triple $ab + ac - bc$ and a photon $a + b - c$. Both of these are nilpotent, as is their sum. The photon is however conflated with the d-distinction, a change in which is mapped notionally to the aforementioned t dimension to achieve a traditional time-like process.

Thus each of the three $\mathcal{T}$'s is a combination of one quaternion component and one photon component. Clearly, $\mathcal{H}$ contains three dimensions - in both the space-like and time-like senses - in the most compact way imaginable. The nilpotence of $\mathcal{H}$ also expresses the existence of a vacuum energy directly.

Note that $\mathcal{H}$'s *form* has 6 components which together generate $2^6 = 64$ sign variants. Of these, 16 are nilpotent and thus Higgs bosons (ie. phases),

$$\mathcal{H} = \{X = \pm ab \pm ac \pm bc \pm ad \pm bd \pm cd \mid X^2 = 0\}.$$

The other 48 square to $\pm abcd$, which we identified in the preceding section as the unit mass carrier; these 48 form the set

$$\mathcal{M} = \{X = \pm ab \pm ac \pm bc \pm ad \pm bd \pm cd \mid X^2 = \pm abcd\}.$$

We interpret the sign of $abcd$ as its rotational orientation in 3+1 space.

We note that for $X \in \mathcal{H}$, $X\,abcd = abcd\,X = \pm X$, but only $abcd\,X = X\,abcd$ for $X \in \mathcal{M}$.

The elements X of $\mathcal{H} \cup \mathcal{M}$ are eigenforms of $abcd$: $|X\,abcd| = |X|$, which in turn define boundaries of $abcd$. That is, we define $\partial_X abcd = X\,abcd$ to be the *boundary* of $abcd$ with respect to X - - in formal analogy to partial differentiation, and with a nod to DeRham's theorem. If also $|X\,abcd| = |X|$, then we further, and oppositely, say that the *co-boundary* of X is $abcd$: $\delta(X) = abcd$. That is, we define the "integral" δ in terms of the "derivative" ∂. [8]

In this way, $abcd$ is the integral of any $X \in \mathcal{H} \cup \mathcal{M}$, since $\delta(wx + yz) = wxyz$. That is, δ is a mass-creation operator with respect to $\mathcal{M}$, and a creation operator generally. Said oppositely, both $\mathcal{H}$ and $\mathcal{M}$ are boundaries of $abcd$, but have very different properties.

We now re-write $\mathcal{H}$ as

$$(1 + abcd)(ab + ac - bc) = \mathcal{H} \tag{1}$$

The factor $1 + abcd$ is a *self-boundary* of $abcd$. Being irreversible, $1 + abcd$ is a time-like operator. This operator is operating on the quaternionic 3d space $ab + ac - bc$, which produces a bosonic potential $\mathcal{H}$.

Thus equation 1 looks like a local version of Einstein's basic GR equation: the time-like aspect of a mass $abcd$, aka. "gravity", operates on a 3d space $ab+ac-bc$ made out of the very same mass aspects, and produces a wave-like, space-like, but inherently dissipative $3 + 1$d potential, aka. the space-time stress tensor. The general form is $\mathcal{H} = (\pm 1 \pm wxyz)(xy + xz + yz)$.

Let X, Y, Z over a, b, c, d and X', Y', Z' over p, q, r, s be two sets of tauquernions written in the above form. Noting that that form commutes, we can write

$$(X + Y + Z)(X' + Y' + Z') = (1 + abcd)(ab + ac - bc)(pq + pr - qr)(1 + pqrs)$$

Thus the mass-mass interaction $(1 + abcd)(1 + pqrs)$ has $|\{(ab + ac - bc)(pq + pr - qr)\}| = 3^2$ spatial connections, cf. Newton's inverse square law.[9]

That is, the dissipative $3d$ tauquernion space can also be seen as the time-like interaction of masses in a reversible 3d quaternion space. If one happens to believe that space is entirely passive, ie. that $(1 + abcd)(1 + pqrs)$ is the whole story, then one arrives at the classical, Newtonian, view of masses in $3d$ space affecting each other mysteriously.

[8] More concisely, $\delta_Q X = \pm Q$ *iff* $\partial_X Q = XQ$ *and* $|XQ| = |X|$. We take ∂ and δ to be elements *of* the algebra - rather than the usual operators *over* the algebra - this being a less sophisticated but more concrete encoding of the same ideas.

[9] However, even though $(ab + ac + bc)(pq + pr + qr)$ is nilpotent, and as well sandwiched between two idempotents, from which it derives, this is *not* a *causal* connection with $(ab + ac - bc)(pq + pr - qr)$ playing the role of Wait, because $(1 + abcd)(1 + pqrs) \neq (1 + abcd)(ab + ac - bc)(pq + pr - qr)(1 + pqrs)$, cf. [11]. That is, in the classical view represented by the form $(1 + abcd)(1 + pqrs)$, space plays *no* causal role. [As well, $(1 + abcd)(1 + pqrs) = (1+pqrs)(1+abcd)$, which commutativity shreds any sequential or localized notion of causality.]

In this context, note that $xy + xz + yz = xyz(x + y + z)$, so electro-magnetism (via photon $x+y+z$) is directly in the picture, namely neatly woven into the $3d$ gravitational space created by the tauquernions. It bears mentioning, though, that the $(\pm 1 \pm wxyz)(xy + xz + yz)$ form obscures the connection to the EPR phenomena that underlie the very existence of *abcd* and the space it both lies in and forms (§5, next).

Recalling eqn. 1

$$(1 + abcd)(ab + ac - bc) = \mathcal{H} = ab + ac - bc + (a + b - c)d \tag{2}$$

which describes matter acting on space, we can multiply through by *abc* to create the *abc*-conjugate form:

$$(1 - abcd)(a + b - c) = a + b - c - (ab + ac - bc)d \tag{3}$$

which describes matter interacting with light. Summing the rhs's of eqns. 2 and 3 (= concurrent occurrence) and re-arranging, we get:

$$(2) + (3) = (a + b - c)(1 + d) \;\; + \;\; (ab + ac - bc)(1 - d)$$

Note that $1 \pm d$ are measurement operators. Recalling the lhs's of eqns. 2 and 3, *Voila*, light interacts with matter in entropic quaternion space (lhs) with resulting effects (rhs) on the light and the space:

$$\begin{array}{c} (1 - abcd)(a + b - c) \\ + \\ (1 + abcd)(ab + ac - bc) \end{array} = \begin{array}{c} (a + b - c)(1 + d) \\ + \\ (ab + ac - bc)(1 - d) \end{array}$$

All four pieces are nilpotent, as are their sums (ie. each side), which indicates that this interaction is an irreversible, ie. thermodynamic, event.

If instead of adding eqns. 2 and 3, we subtract their expanded forms:

$$\begin{array}{c} (1 - abcd)(a + b - c) \\ - \\ (ab + ac - bc)(1 - d) \end{array} = \begin{array}{c} (a + b - c)(1 + d) \\ - \\ (1 + abcd)(ab + ac - bc) \end{array}$$

then both sides simplify to $(a+b-c)-abc(a+b-c)$, a purely electro-magnetic state.

If this seems obscure, it is perhaps well to recall that every expression in the algebra is a Fourier decomposition, and so what is being 'added' are the oscillations of concurrent processes at various frequencies, phases, and dimensionalities.[10] That is, these descriptions are "particulate" only insofar as one can single out some sub-expression that is unitary that one can then try to measure.

5. Entanglement

We now expand on our earlier statement that each $\mathcal{T}_i$ is a quantum mechanical Bell/Magic operator, and that the $\mathcal{T}_j$ and $\mathcal{T}_k$ are the Bell/Magic states. These operators capture quantum entanglement, and are the bread and butter of quantum computing research and practice. For the reader's convenience, Table 1 reviews these as they are usually represented. [In this section, we will refer to QM's causal potential with the symbol Υ.]

We have previously shown [14] that the Bell operator is $ab+cd$ and the Magic operator is its conjugate, $ab-cd$; and that these operators are irreversible due to multiplicative cancellation. Two *q*bits q_A and q_B in classical states $q_A = a-b$ and $q_B = c-d$ define an initial global state $q_Aq_B = (a-b)(c-d) = ac-ad-bc+bd = \mathcal{T}_j + \mathcal{T}_k$.[11] This global state is called "classical" because it namely can be factored ("separated") like this. The Bell and Magic operators entangle such classical states to produce an *e*bit, which, in not being so separable, displays the characteristic EPR properties [15].[12]

*E*bits have the same form as *q*bits except that they are a sum of *bi*vectors, instead of vectors. A *q*bit spinor is a single bivector ab or cd, but an *e*bit spinor is the sum of entangled spinors, eg. $ac+bd$. Like a *q*bit, an *e*bit acts as a single co-exclusion (§7.2), even though it is made out of two *q*bits.

One can only be amazed to find, as Tables 2 and 3 show, that the Bell/Magic operators and the states they generate *also* in fact *exactly cover* $\mathcal{H}$, and thus constitute a *completely different* partitioning and view of $\mathcal{H}$-space, which, let us

[10] Namely, in our example, $a+b-c+ab+ac+ad-bc+bd-cd-abd-acd+bcd$.

[11] Meaning (reading off the signs) $a \wedge \neg b$ and $c \wedge \neg d$, ie. each encoding "zero" (vs. "one": $-a+b$ and $-c+d$). Thus $q_0^A = q_0^B =$ "0" here. This encoding, while conceptually redundant, makes "zero/one" superposition states explicit.

[12] The original labels for *q*bits in [13] were $q_A = a_0 - a_1, q_B = b_0 - b_1$, so $q_Aq_B = a_0b_0 - a_0b_1 - a_1b_0 + a_1b_1$. Therefore $Bell = a_0a_1 + b_0b_1$ and $Magic = a_0a_1 - b_0b_1$, so then $B_0 = -a_0b_0 + a_1b_1$ and $M_0 = a_0b_1 - a_1b_0$, whence one can clearly and explicitly see the entanglement in the redistribution of the a_i and b_j, whence q_A and q_B are no longer *separable*.

basis	*basis state 1*	*basis state 2*	*basis state 3*	*basis state 4*
standard	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
diagonal	$\|0'0'\rangle$	$\|0'1'\rangle$	$\|1'0'\rangle$	$\|1'1'\rangle$
Bell	Φ^+	Φ^-	Ψ^+	Ψ^-
	$\frac{1}{\sqrt{2}}(\|00\rangle+\|11\rangle)$	$\frac{1}{\sqrt{2}}(\|00\rangle-\|11\rangle)$	$\frac{1}{\sqrt{2}}(\|01\rangle+\|10\rangle)$	$\frac{1}{\sqrt{2}}(\|01\rangle-\|10\rangle)$
Magic	$\frac{1}{\sqrt{2}}(\|00\rangle+\|11\rangle)$	$\frac{i}{\sqrt{2}}(\|00\rangle-\|11\rangle)$	$\frac{i}{\sqrt{2}}(\|01\rangle+\|10\rangle)$	$\frac{1}{\sqrt{2}}(\|01\rangle-\|10\rangle)$

Table 1: A summary of quantum mechanical bases in standard notation.

not forget, has a definite $3+1$d cast. Successive application of the Bell/Magic operators produces the corresponding Bell/Magic states. Notice that the states drop from four bivectors ($\mathcal{T}_j + \mathcal{T}_k$) to two bivectors ($-\mathcal{T}'_j$) due to cancellation, and it is this information loss that makes the entanglement thermodynamically irreversible.[13]

The Bell and Magic states are 90^o out of phase,[14] and since the starting state is generally some classical state like $q_A q_B = ac - ad - bc + bd$, which can now be rewritten as $B_3 + M_3$, the multiplicative cancellation occurs due to $M \times Bell = 0$, $B \times Magic = 0$ or $B \times M = 0$. These cancellations mean these states have disappeared from the causal potential Υ, and cannot be reached by any multiplicative operator ("transformation"), but rather only by addition, eg. $M_3 = B_0 - ac$. Recall that addition means concurrency, ie. $-ac$ comes from the outside.

The fact that the Bell and Magic states cannot transit (back) to classical states via multiplication is relevant as well to the $\mathcal{M}^2 = abcd$ states. For example, $\mathcal{M} = ab + ac + ad + bc + bd + cd = Bell + B_1 + M_3$, a mixture of Bell and Magic states. Only by concurrently *adding* new bivectors to the mix can a system exit these cyclical/closed/entangled state spaces. Since all of these states are related via entanglement relationships, we see that "mass" is massively entangled.[15] In the language of EPR, the $\Phi^\pm$ and $\Psi^\pm$ are singletons that represent maximally entangled states and behave as multiple "things" acting as one, with consequent non-local correlations. Mass, once created, is thus stabilized.

Table 4 demonstrates that the states $\mathcal{T}_j$ and $\mathcal{T}_k$ are the Bell and Magic states. [We have shown only two conjugate sets of tauquernions here, but as noted earlier, there are eight. Of the eight, four are related to $\mathcal{T}_i$ and the other four to $\mathcal{T}'_i$. The groups of four are all sign variants of each other such that $\mathcal{H} = \mathcal{T}_i + \mathcal{T}_j + \mathcal{T}_k$, whence $\mathcal{H}^2 = 0$. All of these sets contain all of the Bell/Magic states.]

[13] We note that the Hilbert space version shows reversibility. So far as we know [14], this is the only such deviation.

[14] In general, the state transitions are $B_{(i+1) mod\, 4} = B_i Bell$ and $M_{(i+1) mod\, 4} = M_i Magic$.

[15] In fact, any two conjugate $\mathcal{T}$'s could be named "Bell" and "Magic" operators, and all would otherwise be the same. It's best to just assume that everything is more or less entangled with everything. Consequently, gravity is everywhere.

The complete overlap of $\mathcal{T}$-space and entanglement space means that fundamentally, $q$bits and $e$bits are directly related to, and in fact *are*, the underpinnings of gravity and mass. Fittingly like gravity, the EPR effect is non-polar, since the two ends of the effect are equivalent and of the same valence. The lesson of this reasoning is that irreversible quantum mechanical entanglement establishes the *associative* footings on which, and out of which, gravity constructs its net.

$q_A q_B Bell \ = B_0 = \Phi^+$	$= -ac + bd = -\mathcal{T}_y'$
$B_0 Bell \ = B_1 = \Psi^+$	$= \ ad + bc = \ \mathcal{T}_z'$
$B_1 Bell \ = B_2 = \Phi^-$	$= \ ac - bd = \ \mathcal{T}_y'$
$B_2 Bell \ = B_3 = \Psi^-$	$= -ad - bc = -\mathcal{T}_z'$
$B_3 Bell \ = B_0$	$= -\mathcal{T}_y'$

Table 2: Bell operator and resulting Bell states.

$q_A q_B Magic = M_0$	$= \ ad - bc = \ \mathcal{T}_z$
$M_0 Magic \ = M_1$	$= -ac - bd = -\mathcal{T}_y$
$M_1 Magic \ = M_2$	$= -ad + bc = -\mathcal{T}_z$
$M_2 Magic \ = M_3$	$= \ ac + bd = \ \mathcal{T}_y$
$M_3 Magic \ = M_0$	$= \mathcal{T}_z$

Table 3: Magic operator and resulting Magic states.

$\mathcal{T}_x$	$\mathcal{T}_y$	$\mathcal{T}_z$
Magic	$M_3 = -M_1$	$M_0 = -M_2$
Magic	$M_3 = -M_1$	$M_2 = -M_0$
Magic	$M_1 = -M_3$	$M_0 = -M_2$
Magic	$M_1 = -M_3$	$M_2 = -M_0$

$\mathcal{T}_x'$	$\mathcal{T}_y'$	$\mathcal{T}_z'$
Bell	$B_2 = -B_0$	$B_1 = -B_3$
Bell	$B_2 = -B_0$	$B_3 = -B_1$
Bell	$B_0 = -B_2$	$B_1 = -B_3$
Bell	$B_0 = -B_2$	$B_3 = -B_1$

Table 4: Equivalence of Tauquernions and Bell & Magic operators

6. Dark Matter

In this section and the next two, we move from explanations of the tauquernions and the structures they form to some consequences. Foremost among these is the question of whether the tauquernions have anything to say about dark matter, which we now take up. §7 then describes our information-theoretic analysis of all of our results to that point, and §8 uses this analysis to tell a Bit Bang story.

Other work [11] has shown that the key elements of the standard model - bosons and fermions, three quark families, etc. - are captured by $\mathcal{G}_3$, which is isomorphic to the Pauli algebra $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ via the mapping $\{iab, iac, ibc\}$. In particular, the unitary elements of $\mathcal{G}_3$ all correspond clearly: [16]

Particle	*$\mathcal{G}_3$ element*	*Family size*
primitive distinction	x	3
neutrino family	$x+y+xy$	3
electron family	$xy+xz$	3*
proton family	$x+y+z+xy+xz$	3
neutron=xyz proton	$y-z+xy-xz+yz$	3
photon	$x+y+z$	1
**Eg. the three electron siblings are:*	$xy+xz, xy+yz, xz+yz$.	

The middle column of the table *exhausts* the catalog of unitary ($X^2 = 1$) entities in $\mathcal{G}_3$ and are all familiar, so dark matter is presumably not to be found here. We therefore must look in $\mathcal{G}_4$. The simplest non-trivial unitary element of $\mathcal{G}_4$ is

$$m = a + b + c + d$$

Assuming that m must be related to mass, ie. $abcd$, we now calculate m's co-boundary to $abcd$, which requires that $|\partial_m abcd| = |m|$, and which yields

$$\partial_m abcd = m\, abcd = -abc + abd - acd + bcd$$

so $\partial_m abcd$ is not similar to m, ie. m is not an eigenform of $abcd$. We can though apply the distributive law to the sum of $\partial_A X = B$ and $\partial_B X = A$, whence $\partial_{A+B} X = (A+B)X$, which yields $|(m + m\, abcd)abcd| = |m + m\, abcd|$. So the desired co-boundary is

[16] Primitive distinctions - first row - may not be observable in actuality. Photons - last row - are, of course, nilpotent. Note that the electron projection operator $-1 + xy + xz = x(-x + y + z)$, ie. an x-rotation of a photon; there is a similar factorization of $-1 \pm \mathit{proton}$. See Appendix I for the complete $\mathbb{Z}_3$ $\mathcal{G}_3$ Standard Model.

$$\delta(a+b+c+d-abc+abd-acd+bcd)=abcd$$

We therefore define, in parallel with $\mathcal{H}\cup\mathcal{M}$, the set $\mathcal{D}$,

$$\mathcal{D}=\{(w+xyz)\ +\ (x+wyz)\ +\ (y+wxz)\ +\ (z+wxy)\}$$

which has $2^8=256$ sign variants. $\mathcal{D}$ is our hypothesis for dark matter, and we now investigate its structure and properties.

If one looks at $\mathcal{D}$ from a projective point of view, the 1-vector generators of the algebra are *points* defined by lines/processes that intersect a common plane, ie. are simultaneous with, and *bivectors* are the *directed lines* on that plane that connect these points. In this projective view, w,x,y,z are then the vertices of a tetrahedral volume element with triangular faces $\{wxy,wxz,wyz,xyz\}=\binom{\{w,x,y,z\}}{3}$. We hypothesize that these four triangles correspond to the 4 Planck areas$/ln\ 2=1$ *bit* relationship [20]. Similarly, the $(x+y+z)$-boundary of the triangular face xyz yields the quaternions $\{xy,xz,yz\}$.

Just as $\mathcal{H}\cup\mathcal{M}$, along with 1 and *abcd*, form the largest *even* sub-algebra of $\mathcal{G}_4$, so $\mathcal{D}$ is the largest *odd* sub-algebra. The elements of $\mathcal{D}$ form three subsets, the elements of the first of which all square to quaternionic triplets:

$$\mathcal{D}_q=\{D\in\mathcal{D}\,|\,D^2=xy+xz+yz,\quad x,y,z\in\{a,b,c,d\}\ \}$$

and contains 128 elements. We note that $xyz\,\mathcal{D}_q=\pm1\pm wxyz+\{H,M\}$.

There are also 96 $\mathcal{D}$'s that are 8^{th} roots of unity:

$$\mathcal{D}_u=\{D\in\mathcal{D}\,|\,D^2=(w+x)(y+z)\ \&\ D^8=1\}$$

Note that $(w+x)(y+z)=-(y+z)(w+x)$, ie. they anti-commute, and so the $\mathcal{D}_u$ possess a spinorial quality. One can also multiply D^2 out: $(w+x)(y+z)=(wy+xz)+(wz+xy)$ and see that these are two tauquernion forms (and, simultaneously, separable states). We will see in §7.1 that the $\mathcal{D}_u$ contain a further subdivision of $96=16+80$, indicating the existence of two types of material dark matter. [This time, $xyz\,\mathcal{D}_u=\pm1\pm wxyz+\mathcal{M}$.]

Finally, there are 32 nilpotents D_0, for which $xyz\,D_0=-1+wxyz+\mathcal{H}$:

$$\mathcal{D}_0=\{D\in\mathcal{D}\,|\,D^2=0\}$$

Thus $\{xyz\,\mathcal{D}\}=\{-1+wxyz+\mathcal{H}\cup\mathcal{M}\}$, ie. normal matter and dark matter can be understood as being 3-dimensionally perpendicular to each other. Finally, $128+96+32=256$, whence $\mathcal{D}=\mathcal{D}_q\cup\mathcal{D}_u\cup\mathcal{D}_0$.

- - -

The fact that $\{xyz\,\mathcal{D}\} = \{-1 + wxyz + \mathcal{H} \cup \mathcal{M}\}$ and therefore that the elements of $\mathcal{D}$ and $\mathcal{H} \cup \mathcal{M}$ can be rotated into each other allows a further analysis.

Let for example $\mathcal{D}_a = -a - bcd$; $\mathcal{D}_b = b + acd$; $\mathcal{D}_c = c - abd$ $\mathcal{D}_d = d + abc$, and define generally $D = \mathcal{D}_a + \mathcal{D}_b + \mathcal{D}_c + \mathcal{D}_d$ such that $D \in \mathcal{D}$. In this example, $D^2 = -bc + bd - cd \in \mathcal{D}_q$. Now construct their multiplication table, ie. D_q^2:

$D_q \times D_q$	$\mathcal{D}_a = -a - bcd$	$\mathcal{D}_b = b + acd$	$\mathcal{D}_c = c - abd$	$\mathcal{D}_d = d + abc$
$\mathcal{D}_a$	0	$ab + cd$	$ac - bd$	$ad + bc$
$\mathcal{D}_b$	$-ab + cd$	0	0	0
$\mathcal{D}_c$	$-ac - bd$	0	0	0
$\mathcal{D}_d$	$-ad + bc$	0	0	0

The sum of the $\mathcal{D}_a$ row is namely $\mathcal{D}_a\mathcal{D}_b + \mathcal{D}_a\mathcal{D}_c + \mathcal{D}_a\mathcal{D}_d \in \mathcal{H}'$, and anti-commutatively, the sum of the $\mathcal{D}_a$ column is $\mathcal{D}_b\mathcal{D}_a + \mathcal{D}_c\mathcal{D}_a + \mathcal{D}_d\mathcal{D}_a \in \mathcal{H}$. That is, $D_q^2 = \mathcal{H} + \mathcal{H}'$! This holds for all elements of $\mathcal{D}_q$ - *all* such tables contain zeroes except for one element each from $\mathcal{H}$ and $\mathcal{H}'$, and thus each element of $\mathcal{D}_q$ harbors the potential for both $\mathcal{H}$ and $\mathcal{H}'$ and so a complete set of tauquernions.[17]

In contrast, the corresponding tables for elements of $\mathcal{D}_0$ contain only zeroes; and the tables for $\mathcal{D}_u$ all look like this one:

$D_u \times D_u$	$\mathcal{D}_a = a + bcd$	$\mathcal{D}_b = b + acd$	$\mathcal{D}_c = c + abd$	$\mathcal{D}_d = d + abc$
$\mathcal{D}_a$	0	$-ab - cd$	0	$ad - bc$
$\mathcal{D}_b$	$ab - cd$	0	$ad - bc$	0
$\mathcal{D}_c$	0	$ad + bc$	0	$-ab - cd$
$\mathcal{D}_d$	$ad - bc$	0	$-ab + cd$	0

wherein we see that only two out of the three tauquernion forms appear, doubled, and including conjugates; the table sums to $ab + cd - ad + bc = (a - c)(b - d)$, where again there is a spinorial aspect (and two separable $q$bits). The missing tauquernion forms can be recovered from the products of the others, so $D_u \times D_u$, like $D_q \times D_q$, harbors an alternative pathway to $\mathcal{H} \cup \mathcal{M}$.

A rather different view emerges when one realizes that most of the partial products $(w + xyz)(x + wyz)$ in fact generate $\mathcal{T}$'s, and it is only their signs and sums in the full 4-way form that generate the three different outcome $\mathcal{D}$'s. Thus $\mathcal{D}_0^2$'s *five* $\mathcal{T}$'s sum to zero (three $\mathcal{T}$'s are identical and the other two complementary),

[17] Since $1{+}3 \leq 4$, dark patches can connect smoothly, and, since the algebra is self-consistent, this connection must be compatible with that of $\mathcal{H} \cup \mathcal{M}$.

$\mathcal{D}_u^2$'s *four* $\mathcal{T}$'s sum to $(w+x)(y+z)$, and $\mathcal{D}_q^2$'s *three* $\mathcal{T}$'s sum to a quaternion triple (three xy's are identical). So there are a *lot* of $\mathcal{T}$'s floating around in the soup. Note in this connection that, like the individual $\mathcal{T}$-components of $\mathcal{H}$ and $\mathcal{M}$, each of the $\mathcal{D}_k$ and sums thereof satisfy $\delta\mathcal{D}_k = abcd$.

Finally, these $\mathcal{T}$'s are also entangled states, so (via xyz-rotation) all of the elements of $\mathcal{D}$ are also entangled, although it seems that this is indirect.

Summarizing, like $\mathcal{H}$ and $\mathcal{M}$, the elements of $\mathcal{D}$ *also* can interact to form space and matter, but more indirectly. A key issue is the energies at which $w + xyz$ and $\mathcal{D}$ form, and closely related is the question of what role the pathways from $\mathcal{D}$ to $\mathcal{H} \cup \mathcal{M}$ play.

A pertinent question at this point is, How do the elements of $\mathcal{D}$ interact with light? We have identified xyz as the carrier of charge [11], but that is in the context that $\delta(x+y+z+xyz(x+y+z)) = xyz$, where $xyz(x+y+z) = xy+xz+yz$ is the spinorial basis of the magnetic effect, and x, y, z each "$\frac{1}{3}$ electrical charge". This context is missing from both $\mathcal{H} \cup \mathcal{M}$ and $\mathcal{D}$. So, on this basis, one should not expect much of an electro-magnetic interaction with either of them (and indeed, 3d space is indifferent to electro-magnetism).

On the other hand, $\mathcal{D}$'s four xyz terms still have spin, even if it isn't identifiable any more as "charge". This spin could nevertheless conceivably retain electric charge's like-sign repulsive property, and so could be advanced as a contributor to the vacuum energy. We also note that H can be rewritten $(w-xyz)(x+y-z)$. However, re xyz (which squares to -1 and hence is 'polar'), where there's a 'plus' there's a 'minus', which polarity opens the door for (eg.) dark "ionic cluster" formation and the like, a possibility that can at this point only be speculation. Finally, $\mathcal{D}_0$ and $\mathcal{D}_q$, both being roots of zero, will both presumably contribute to the vacuum energy.

Howsoever, the fact that there now is a detailed mechanism in hand should simplify the task of finding a viable way to detect dark matter generally.

7. Information Content and Kind

We now embark on exact calculations of the information content (and its transformation) of expressions in the algebra. The overall picture is a "Bit Bang" modelled as the algebraic expansion $\mathcal{G}_0 \to \mathcal{G}_1 \to \mathcal{G}_2 \to \mathcal{G}_3 \to \mathcal{G}_4$, which expansion is driven by entropy creation via the conversion of information from space-like (non-Shannon) to time-like (Shannon) form.

Section §7.1 calculates the numerical information-theoretic skeleton of our $\mathbb{Z}_3$ $\mathcal{G}_4$ algebra, which is possible because of its finiteness and relatively small size: $3^{16} \approx 43$ million expressions. Since the algebra *is* the phase space, this *exact* (!) numerical skeleton has cosmological implications that we pursue in §8.

Section §7.2 then describes the computational mechanisms that define and create the aforementioned space-like, non-Shannon information. [Our use of the term "non-Shannon information" is distinct from, but consistent with, a like-sounding entropy-related term, "non-Shannon-type inequalities".]

In this section we refer to $G = \{1, a, b, c, \ldots, ab, ac, \ldots, abc, \ldots, \ldots\}$ rather than $\mathcal{G}$ because we are referring specifically to *actual instantiated elements*, though these of course also belong to the *abstract* geometric algebra $\mathcal{G}$.

7.1 Calculating Information Content

The formal concept of *information* is due to Claude Shannon (1948), who defined the information content $\mathcal{I}$ of an event x as

$$\mathcal{I}(x) = -lg\, p_x$$

where p_x is the probability of occurrence of the event x, and lg is the logarithm to the base 2. Thus, as is well known, the more improbable the event, the greater its information content. The import of this definition for us is best understood with the example of an ***if-then-else****-type* decision. The form [11]

$$X(\,1 + \langle\, -1 - a, \pm\mathsf{a}\rangle\,) \;\;+\;\; Y(\,1 + \langle\, -1 + a, \pm\mathsf{a}\rangle\,)$$

describes the computation *if* a *then* X *else* Y, where the brackets $\langle\,,\rangle = \pm 1$ indicate the inner product of the idempotent measurement probe $-1 \pm a$ with an entity $\pm\mathsf{a}$ in the surround, and $+$ indicates as usual the concurrency of the processes $X, Y \in G$. Here we see that a *static* bit of information - encoded in the $\pm$ state of a - is converted into the *motion* [state change] of *one* of the processes X or Y, since one of the two expressions *will* yield zero and the other minus one (minus because X (or Y) now changes state). Note particularly that *the information is consumed*: a has been changed by the measurement and no copies made. One correctly concludes that a binary decision *costs* one bit of information.[18]

Applying this to G_n, this means that a measurement sequence that would locate some entity $\in G_n$ having an information content of m bits would require m such nested *if*'s. Furthermore, this decision process transforms the static *space-like* information contained in the current state of G_n into dynamic *time-like* information at an exchange rate of $1:1$. It is this transformation (on a massive scale) that constitutes our expanding time-like universe.

This transformation is fundamentally entropic in character. Because the algebra is finite, we can calculate the probability of occurrence of an expression, and so we can know its information content. Knowing that, we can follow the entropy trail - loss of information - and make predictions about what further transformations will occur.

We therefore now embark on the calculation of the information content, measured in *bits*, of every element of G_n, $n = 0, 1, 2, 3, 4$. This is an *exact* calculation, since it is based on pure combinatorics and resulting integer ratios.

The binary nature of our algebra allows us to fully expand the combinatorial content of any given expression in the fashion of a "truth table". Below we show the tables for ab, abc, and $abcd$. Beneath the tables are vectors of the respective *result* (rightmost) columns; *these result vectors are the basis for our information content analysis.*

[18] Notice, by the way, how the 1-dimensional 180^o opposition between X and Y as coded in $\pm$a becomes a conjugate (90^o) opposition, $-1-a$ vs. $-1+a$, in the translation from the sequential to the concurrent view.

a	b	ab
$-$	$-$	$+$
$-$	$+$	$-$
$+$	$-$	$-$
$+$	$+$	$+$

a	b	c	abc
$-$	$-$	$-$	$-$
$-$	$-$	$+$	$+$
$-$	$+$	$-$	$+$
$-$	$+$	$+$	$-$
$+$	$-$	$-$	$+$
$+$	$-$	$+$	$-$
$+$	$+$	$-$	$-$
$+$	$+$	$+$	$+$

a	b	c	d	$abcd$
$-$	$-$	$-$	$-$	$+$
$-$	$-$	$-$	$+$	$-$
$-$	$-$	$+$	$-$	$-$
$-$	$-$	$+$	$+$	$+$
$-$	$+$	$-$	$-$	$-$
$-$	$+$	$-$	$+$	$+$
$-$	$+$	$+$	$-$	$+$
$-$	$+$	$+$	$+$	$-$
$+$	$-$	$-$	$-$	$-$
$+$	$-$	$-$	$+$	$+$
$+$	$-$	$+$	$-$	$+$
$+$	$-$	$+$	$+$	$-$
$+$	$+$	$-$	$-$	$+$
$+$	$+$	$-$	$+$	$-$
$+$	$+$	$+$	$-$	$-$
$+$	$+$	$+$	$+$	$+$

$$[+--+] \quad [-++-+--+] \quad [+--+-++--++-+--+]$$

As an example, we take the vector for abc and add to it $+\mathbf{1}$ and $-\mathbf{1}$:

$$\begin{array}{r} abc = [-\ ++-+--+] \\ +\mathbf{1} = [++++++++] \\ \hline [\cdot\ --\ \cdot - \cdot\ \cdot -] \end{array} \qquad \begin{array}{r} abc = [-++-+--+] \\ -\mathbf{1} = [--------] \\ \hline [+\ \cdot\ \cdot\ +\cdot++\cdot] \end{array}$$

Note that the *pattern* of symbols is the same for abc and the two sums, the only difference being that in abc, the two symbols that appear are $+$ and $-$, whereas in the sums the two symbols are $\cdot$ and $-$, and $+$ and $\cdot$, respectively (recall that $\cdot$ symbolizes zero). But the *ratios* are the same: here, four of each and *no* third symbol. And if you think about it, this proportionality will always hold - all that happens with the summing of abc with $\pm\mathbf{1}$ is that one so-to-speak rotates a three-symbol mapping vector $[\cdot\ ,+,-]$ first to $[-\ ,\cdot\ ,+]$ and then to $[\ +,-\ ,\cdot]$: *the proportions will therefore always be the same.*

Argument. A *pattern encoding* consists of a 3-tuple (#0's, #1's, #-1's), which forms a *signature* of the vector's structure. Suppose we have the pattern vector $(2, 2, 4)$ and imagine a (minimal) decision tree - think nested *if*'s - that identifies any expression having this pattern. Then the amount of information embedded implicitly in the tree's decision points is the measure of the tuple's information content. The three symbols are interchangeable because the tree's *form* (the structure of the search space) is indifferent to which symbols lie at its leaves.

Since the ratios are invariant under exchange of symbols, the counts can appear in any order, so we just sort the tuples numerically.

This symbol-invariance implies that $\pm abcd$ and $\pm 1 \pm abcd$ all have the same information content. Since the latter form defines a measurement on the former, and these two therefore *should* be the same, this is comforting. For this and similar reasons, we think that any classification scheme (cf. *binning*, below) must subscribe to the collapsing of $0, 1, \tilde{1}$ into one signature.[19]

This all means that we can classify *every* expression in the algebra in terms of its result-vector's signature. We will soon see that these informational classifications exactly match the $\mathcal{H}, \mathcal{M}, \mathcal{D}$, bosonic, and unitary particle structures previously discussed.

Since a polynomial $\in \mathcal{G}_n$ has maximally $|G_n| = 2^n$ mutually orthogonal terms, and their coefficients can be one of $0, 1, -1$, we get the set S, of size $|S| = 3^{2^n}$, which covers all of the possible expressions in G. With S in hand, we can count how many times k each pattern X occurs, and we can then divide k by $|S|$ to get the probability p of X's occurrence:

$$p_X = \frac{k}{3^{|G_n|}}$$

If $k = 1$, then is there is but one single occurrence of X in S, so p_X would be minimal, but this actually can't happen - the best you can do is the three scalar constants, $0, 1, \tilde{1}$, where $k = 3$.

From the other end of the microscope, a minimal X requires the full measure of the information in S in order to be identified and isolated. That is, the information content $\mathcal{I}$ of an expression $X \in G$ is

$$\mathcal{I}(X) = -lg\ p_X = -lg\ \frac{k}{3^{|G_n|}} = \ lg\ \frac{3^{2^n}}{k} \quad bits$$

X's information content is thus a function of how many other X's share its signature, and the size of the space it occurs in.

An obvious application of this is to ask, What is the information content of some particle P, having in mind the fact [20] that 1 $bit = 4$ *Planck areas* $/ln\,2$ ($\approx 10^{-66}\,cm^2$).

Thus, for example, a single Higgs boson $H = (1 + wxyz)(xy + xz + yz) = xy + xz + yz + wx + wy + wz$ exists in 16 states out of the 64 possible in the form. Its information content is therefore

[19] $\mathcal{V}oid$ cannot have its own category because, by definition, it has no properties by which it might be so categorized, including the property of having no properties. $\mathcal{V}oid$ can first become manifest in the distinction $[1, \tilde{1}]$.

$$\mathcal{I}(H) = lg\,\frac{3^{16}}{16} = 21.3594000\ bits$$ [20]

The next step, the conversion of *bits* to *Gev*, turns out to be unexpectedly complicated, and is our current focus. The final paper will hopefully contain this result for $\mathcal{H}, \mathcal{M}, \mathcal{D}$, and all the rest too.

Of interest equal to individual particles, however, is the picture painted with the broader brush of the signatures and bin counts themselves.

Table 5 lists the information content, calculated in this broader way, of relevant elements of G_n. Because rarity/information is relative to the size of the space, the measure of (say) ab is 2.17 bits in G_2, 7.29 bits in G_3, and 18.9 bits in G_4. But at the same time, all of a, ab, abc, and $abcd$, at any given level, have the *same* measure, since their uniqueness stays proportional to n; note that namely these *also* have the highest information content after $0, 1, \tilde{1}$. In general, the lower the bit value, the larger the family of entities having that count, and oppositely, the higher the count, the smaller the family. We now explore this a little more.

The function $bitsN(X) = lg\,\frac{3^{2^N}}{count(X's)}$ calculates the information content of $X \in G$ relative to G_N.

Then, *re* G_0, the three scalar constants $0, 1, -1$ are all *known* and occupy the entire space, which is of size $3^{2^0} = 3$ states, one each for $\{0, 1, -1\}$,

- So each occurs with probability $p = \frac{1}{3} \mapsto lg\,3 = 1.58$ bits, *but*
- *Known* means $bits0(0) = bits0(1) = bits0(-1) = lg\,\frac{3}{3} = lg\,1 = 0$
- So G_0 actually contains no information.

In G_1 there are $3^{2^1} = 9$ states, three for G_0's scalars, $\in (0, 0, 2)$, and $2 + 4 = 6$ more for $\pm a$ and $\pm 1 \pm a$, both $\in (0, 1, 1)$:

- The scalar constants are *known*, and so they contain no information, but nevertheless occupy three slots in the state space $\Rightarrow bits1(1) = \lg 3 = 1.58$ bits.[21] [It is a mod-3 coincidence that the numbers for G_0 and G_1 are the same.]
- 1-vectors occupy the remaining states in G_1, so $bits1(\pm a) = \lg \frac{9}{6} = 0.58$ $= bits1(\pm 1 \pm a)$.

[20] This result differs from Table 5 (below) because we have ignored the other members of its bin, (4,4,8). Also, we don't know what the experimentalists are actually measuring - perhaps we should have calculated $\pm 1 \pm H$, etc.

[21] Recall that $0, 1, -1$ all map to the same pattern, whence 3 and not 1.

- The net result is that exactly 1 (classical) bit of information is encapsulated in the structure a: $bits1(1) - bits1(\pm a) = 1.00$.

For G_2, the algebra of pure $q$bits:

- The scalar constants are *known*, but occupy state space: $bits2(1)= \lg(\frac{81}{3}) =$ 4.75 bits.$^{(ditto)}$
- Here is a smallest addressable state: $(1-a)(1-b) = 1-a-b+ab \in (0,1,3)$ $\mapsto bits2((1-a)(1-b)) = lg(\frac{81}{24}) = 1.75$ bits, corresponding to a single row of the form's "truth table". The 24 count comes from the 2^4 sign variants of $1-a-b+ab$ plus the 2^3 sign variants of $a+b+ab$.
- In §7.2, we show how it is that simple concurrency, $a+b$, *mere concurrent existence*, contains and encodes information. Here we just calculate: $bits2(a) = 2.17 = bits2(b)$, $bits2(ab) = 2.17$, $bits2(a+b) = 1.17$ bits,
- Whence $bits2(ab) - bits2(a+b) = 1.00000000$, where we show in the 0's the number of significant digits that actually are available in these (exact) calculations; we show rounded values otherwise.

In G_4:

- Let $m = a+b+c+d$, whence $D = m + m\,abcd$ and $D^2 = 0$. As shown in Table 5, $\mathcal{D}_0 \in (4,4,8)$ and each D contains 5.53 bits of information.
- But $abc\,D = -1+ab-ac+ad+bc+bd+cd+abcd$ computes to 6.87 bits (not shown). One does not expect a reversible operator like abc to change the information content of an entity.
- The explanation is that the rotation by abc changes the *signature bin* that the expression falls into, and the new bin, namely $(2,6,8)$, has fewer members, and so the information content is higher. "It's not the rotation's fault." [We will exploit this phenomenon in our Bit Bang story in §8.]
- In Table 5, there are two examples of binnings that further differentiate the 3-signature - $(a+b+c)d$ and $\mathcal{M}_2$ are both $\in (4,6,6)$, yet their bit-measures differ, 12.1 vs. 7.08, and again $a+bcd$ and $\mathcal{D}_0$ are both $\in (4,4,8)$, but their measures are 15.1 vs. 5.53.

Particle/Form	*Vector ($\mathcal{G}_3$ and $\mathcal{G}_4$ samples)*	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$
$\mathcal{G}_0$					
$\mathcal{V}oid \mapsto \mathbf{0}$ *is*	$[\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot]$ $\in (0,0,8),0$	1.58	4.75	11.1	23.8
$\pm\mathbf{1}$ *are*	$[\pm\pm\pm\pm\pm\pm\pm\pm]$ $\in (0,0,8),0$	1.58	4.75	11.1	23.8
$\mathcal{G}_1$					
a $\pm$*exist*	$[----++++]$ $\in (0,4,4),1$	0.58	2.17	7.29	18.9
$1-a$ *(measure)*	$[----\cdot\ \cdot\ \cdot\ \cdot]$ $\in (0,4,4),1$	0.58	2.17	7.29	18.9
Row0 $(1-w)\dots(1-z)$	$\in (0,1,1),(0,1,3),(0,1,7),(0,1,15)$	0.58	1.75	7.09	18.8
$\mathcal{G}_2$					
ab $\pm$*spin*	$[++----++]$ $\in (0,4,4),1$	–	2.17	7.29	18.9
$a+b$ *co-occ*	$[++\cdot\ \cdot\ \cdot\ \cdot--]$ $\in (2,2,4),2$	–	1.17	4.70	15.1
$a+b+ab$ ν	$[------\cdot\ \cdot]$ $\in (0,2,6),3$	–	1.75	5.29	15.6
$a+ab$ W,Z †	$[\cdot\ \cdot++\cdot\ \cdot--]$ $\in (2,2,4),2$	–	1.17	4.70	15.1
$1+ab$	$[--\cdot\ \cdot\ \cdot\ \cdot--]$ $\in (0,4,4),1$	–	2.17	7.29	18.9
$\mathcal{G}_3$					
abc $\pm$*charge*	$[-++-+--+]$ $\in (0,4,4),1$	–	–	7.29	18.9
$a+bc$ *quark*	$[\cdot++\cdot-\cdot\ \cdot-]$ $\in (2,2,4),2$	–	–	4.70	15.1
$ab+ac$ e	$[-\cdot\ \cdot++\cdot\ \cdot-]$ $\in (2,2,4),2$	–	–	4.70	15.1
$a+b+c+ab+ac$ p	$[----\cdot++-]$ $\in (1,2,5),5$	–	–	2.70	11.5
$a+b+c$ γ	$[\cdot--+-++\cdot]$ $\in (2,3,3),3$	–	–	3.29	12.1
$ab+ac+bc$ *3-space*	$[\cdot------\cdot]$ $\in (0,2,6),3$	–	–	5.29	15.6
$\mathcal{G}_4$					
$abcd$ $+$*mass*	$[+--+-++--++-+--+]$ $\in (0,8,8),1$	–	–	–	18.9
$1-abcd$	$[\cdot--\cdot-\cdot\ \cdot--\cdot\ \cdot-\cdot--\cdot]$ $\in (0,8,8),1$	–	–	–	18.9
q_Aq_B *2 qbits*	$[\cdot\ \cdot\ \cdot\ \cdot\ \cdot+-\cdot\ \cdot-+\cdot\ \cdot\ \cdot\ \cdot\ \cdot]$ $\in (2,2,12),4$	–	–	–	14.1
$a+b+c+d$	$[-++\cdot+\cdot\ \cdot-+\cdot\ \cdot-\cdot--+]$ $\in (5,5,6),4$	–	–	–	10.1
$(a+b+c)d$	$[\cdot\ \cdot+-+--++--+-+\cdot\ \cdot]$ $\in (4,6,6),3‡$	–	–	–	12.1
$\mathcal{M}_1$ *(16/64) proto-mass*	$[\cdot\ \cdot\ \cdot+\cdot++\cdot\ \cdot++\cdot+\cdot\ \cdot\ \cdot]$ $\in (0,6,10),6$	–	–	–	13.1
$\mathcal{M}_2$ *(32/64) proto-mass*	$[++-\cdot-\cdot-++-\cdot-\cdot-++]$ $\in (4,6,6),6‡$	–	–	–	7.08
$\mathcal{H}$ *(16/64)* *Higgs*	$[-\cdot++\cdot-+--+-\cdot++\cdot-]$ $\in (4,6,6),6$	–	–	–	7.08
$Bell = ab+cd = \mathcal{T}'$	$[-\cdot\ \cdot-\cdot++\cdot\ \cdot++\cdot-\cdot\ \cdot-]$ $\in (4,4,8),2$	–	–	–	15.1
$Magic = ab-cd = \mathcal{T}$	$[\cdot--\cdot+\cdot\ \cdot++\cdot\ \cdot+\cdot--\cdot]$ $\in (4,4,8),2$	–	–	–	15.1
$B_0 = -ac+bd$	$[\cdot+-\cdot+\cdot\ \cdot--\cdot\ \cdot+\cdot-+\cdot]$ $\in (4,4,8),2$	–	–	–	15.1
$M_0 = ad-bc$	$[\cdot+-\cdot-\cdot\ \cdot++\cdot\ \cdot-\cdot-+\cdot]$ $\in (4,4,8),2$	–	–	–	15.1
$a+bcd$ *dark*	$[+\cdot\ \cdot+\cdot++\cdot\ \cdot--\cdot-\cdot\ \cdot-]$ $\in (4,4,8),2‡$	–	–	–	15.1
$\mathcal{D}_0$ *dark*	$[-\cdot-\cdot\ \cdot-\cdot+-\cdot+\cdot\ \cdot+\cdot+]$ $\in (4,4,8),8‡$	–	–	–	5.53
$\mathcal{D}_q$ *dark*	$[++-\cdot++--++--\cdot+--]$ $\in (2,7,7),8$	–	–	–	6.87
$\mathcal{D}_u$ *(80/96)* *dark*	$[--\cdot\ \cdot\ \cdot\ \cdot-+-+\cdot\ \cdot\ \cdot\ \cdot++]$ $\in (4,4,8),8$	–	–	–	5.53
$\mathcal{D}_u$ *(16/96)* *dark*	$[+\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot-]$ $\in (1,1,14),8$	–	–	–	15.9

Table 5: Information content (in *bits*) of principal $\mathcal{G}_n$ forms. † Tentative. ‡ See text.
Note palindromes.

These examples show that information content values like those in Table 5 are sensitive to the binning algorithm that is used. Fortunately, whatever the binning, the results will always be consistent because the underlying population is the same.

Our general-purpose binning algorithm (used in Table 5) first applies the 3-pattern signature, and then further bins together only those expressions having the same number of non-scalar terms.[22] Therefore, co-occurrences/$q$bits $x + y$, electrons $xy + xz$, and quarks $x + yz$, which already have the same signature, will still bin together. Thus, the numbers in Table 5 and its cousins will always be indicative rather than definitive, since how one bins is determined by which interaction-classes one is interested in.

There are other interesting things in Table 5: the information content of space as described by classical quaternions is 3.3 bits smaller than that of matter $(15.6 - 18.9)$. Photons $(a + b - c)$ and their confounding $(a + b - c) * d$ have the same measure, 12.1, which is rather larger than $\mathcal{H}$'s 7.08, which contains them. There are apparently *two* forms of proto-mass $\mathcal{M}$ (13.1 vs. 7.08), and we note that the former is a sparse $+1$ variant. Singletons always have the highest bit value after the scalars, even more than two classical $q$bits $q^A q^B$. But then, given their spin, they *are* bits yo. Finally, note that the Bell/Magic states, $\mathcal{D}$, quarks, and electrons all have the same measure, 15.1, only slightly less likely than light, 12.1; and versus the rather more likely $\mathcal{H}$ and $\mathcal{M}$ at 7.08 bits.

Howsoever, as the expansion proceeds - - $\mathcal{G}_1 \to \mathcal{G}_2 \to \mathcal{G}_3 \to \mathcal{G}_4$ in Table 5 - - Ψ's information content shrinks as the information in $3+1$d gets denser and denser. For example, the two classical bits q^A, q^B use 4 spinors and 14.1 bits to encode 1 $e$bit - time-like stability costs! Matter itself is only slightly denser at 18.9 bits per: frozen potential (because actualized), robbed of its variability through loss of degrees of freedom. This is the fate of the space-like non-Shannon information that is converted, as the expansion of the universe, into time-like Shannon information.

We pursue this entropic expansion in a cosmological setting in §8. Before doing so, we introduce and define the concept of non-Shannon information, and show how this builds structure.

7.2 Non-Shannon information

There is a subtle paradox - concerning *kinds* of information - that we must deal with before going further. Shannon's concept of information, as we have seen, can be viewed as a descent into a binary tree from root to leaf, where at

[22] This increases the number of bins from 10 to 14 for G_3, and from 30 to 86 for G_4. For example, in the text just above, $(a + b + c)d \in ((4, 6, 6), 3)$, $\mathcal{M} \in ((4, 6, 6), 6)$, $a + bcd \in ((4, 4, 8), 2)$ and $\mathcal{D}_0 \in ((4, 4, 8), 8)$. All 43 million expressions were binned.

each branch point, one bit is consumed in the choosing of one path versus the other. Two points should be noted: (1) the (*information represented by the*) bits are(*is*) *consumed* and converted into the motion/advance of the descent-process; and (2) *action* is what this is all about... this *sequential* process is blind to context, and sees only its own (namely causal) point of view. The process concept, here exemplified, *is* sequence and action, combined. Thus Shannon's view of information is purely *time-like*.

It is difficult to see how Shannon's definition misses anything out, and yet ... it does. There *is* a kind of information that falls beyond it, namely the information of *concurrent existence*, what we call *non-Shannon information*. The following *Coin Demonstration* makes the argument.

Act I. *A man stands in front of you with both hands behind his back. He shows you one hand containing a coin, and then returns the hand and the coin behind his back. After a brief pause, he again shows you the same hand with what appears to be an identical coin. He again hides it, and then asks, "How many coins do I have?"*

Understand first that this is not a trick question, or some clever play on words - we are simply describing a particular and straightforward situation. The best answer at this point then is that the man has "at least one coin", which implicitly seeks *one bit* of information: two possible but mutually exclusive states: *state1* = "one coin", and *state2* = "more than one coin".

One is now at a decision point - *if* one coin *then* X *else* Y - and only one bit of information can resolve the situation. Said differently, when one is able to make this decision, one has *ipso facto* received one bit of information.

Act II. *The man now extends his hand and it contains two identical coins.*

Stipulating that the two coins are in every relevant respect identical to the coins we saw earlier, we now know that there are *two* coins, that is, *we have received one bit of information*, in that the ambiguity is resolved. We have now arrived at the dramatic peak of the demonstration:

Act III. *The man asks, "Where did that bit of information come from?"*

Indeed, where *did* it come from??! The bit originates in the *simultaneous presence* of the two coins - their ***co-occurrence*** - and encodes the now-observed *fact* that the two *processes,* whose states are the two coins, respectively, do not exclude each other. [23]

Thus, there is information in (and about) the environment that *cannot* be acquired sequentially, and true concurrency therefore cannot be simulated by a

[23] Cf. Leibniz's indistinguishables, and their being the germ of the concept of space: simultaneous events, like the presence of the two coins, are namely indistinguishable in time.

Turing machine. Penrose concluded in [18] that Turing machines cannot simulate quantum mechanics. Both Turing and Penrose consider the case $f \parallel g$, meaning execute the non-interacting processes f and g in parallel (and harvest their results when they end). Clearly one gets the same results whether one runs f first $(f; g)$ or g first $(g; f)$, or simultaneously, $f \parallel g$. In this *functional* view of computation, the only difference is wall-clock time. The Coin Demonstration is not about these cases *at all*, but rather asks, Can f *exist* simultaneously with g, or do they *exclude* each other's existence? This is the fundamental distinction that we draw.

More formally, we can by definition write $a + \tilde{a} = 0$ and $b + \tilde{b} = 0$, meaning that (process state) a excludes (process state) $\tilde{a}$, and similarly (process state) b excludes (process state) $\tilde{b}$. [24] Their *concurrent* existence can be captured by adding these two equations, and associativity gives two ways to view the result. The first is

$$(a + \tilde{b}) + (\tilde{a} + b) = 0$$

which is the usual excluded middle: if it's not the one (eg. that's $+$) then it's the other. This arrangement is convenient to our usual way of thinking, and easily encodes the traditional *one/zero* (or $1/\tilde{1}$) distinction.[25] The second view is

$$(a + b) + (\tilde{a} + \tilde{b}) = 0$$

which are the two superposition states: either both or neither.

The Coin Demonstration shows that *by its very existence*, a 2-co-occurrence like $a+b$ contains one bit of information. Co-occurrence relationships are *structural*, ie. *space-like*, by their very nature. Such bits, being space-like, are the source of non-Shannon information.

[Cf. Table 5, this information is twice that of a or b alone *in* G_1, but $2.17-1.17 =$ 1 bit less than a, b or ab *in* G_2.]

Act IV. *The man holds both hands out in front of him. One hand is empty, but there is a coin in the other. He closes his hands and puts them behind his back. Then he holds them out again, and we see that the coin has changed hands. He asks, "Did anything happen?"*

[24] This is the logical bottom, and so there are no superpositions of $a/\tilde{a}$ and $b/\tilde{b}$: they are 1d exclusionary distinctions . Superposition first emerges at level 2 with ab via the distinction *exclude* vs. *co-occur*.

[25] Since $\tilde{x}$ is not the same as $0x$, an occurrence $\tilde{x}$ is meaningful; in terms of sensors, $\tilde{x}$ is a *sensing* of an externality x, not x itself.

This is a rather harder question to answer. To the above two *con*current *exclusion*ary processes we now apply the *co-exclusion inference*, whose opening syllogism is: *if* a excludes $\tilde{a}$, *and* b excludes $\tilde{b}$, *then* $a + \tilde{b}$ excludes $\tilde{a} + b$ (or, conjugately, $a + b$ excludes $\tilde{a} + \tilde{b}$).... This we have just derived.

The inference's conclusion is: *and therefore, ab exists*. The reasoning is that we can logically replace the two one-bit-of-state processes a, b with one two-bits-of-state process ab, since what counts in processes is sequentiality, not state size, and exclusion births sequence (here, in the form of alternation). That is, the existence of the two co-exclusions $a + \tilde{b} \mid \tilde{a} + b$ and $a + b \mid \tilde{a} + \tilde{b}$ contains sufficient information for ab to be able to encode them, and therefore, logically and computationally speaking, ab can rightfully be instantiated. We write $\delta(a+\tilde{b}) = ab = -\delta(\tilde{a} + b)$ and $\delta(a + b) = ab = -\delta(\tilde{a} + \tilde{b})$. A fully realized ab is, we see, comprised of two *conjugate* co-exclusions, a sine/cosine-type relationship.

We can now answer the man's question, *Did anything happen*? We can answer, "Yes, when the coin changed hands, the state of the system rotated 180^o: $ab(a+\tilde{b})ba = \tilde{a}+b$." We see that one bit of information ("something happened") results from the alternation of the two mutually exclusive states.

With the co-exclusion concept in hand, we can now add a refinement to the idea of co-occurrence. Recall that S is the space of all imaginable expressions in $\mathcal{G}$. But, thinking now computationally, this means that they are all "there" at the same time! That is, *S is the space of superpositions*, of all *imaginable* co-occurrences of elements of $\mathcal{G}$ *all at the same time*; whereas G is the space of *actually occurring* (but still space-like) entities, which means no co-exclusionary states allowed. When things move from S to G, superposition is everywhere replaced by reversible alternation, ie. G is a sub-space of S.

Co-exclusions, being superpositions, thus live exclusively in S, whereas co-occurrences can exist in both S and G, though their objects are slightly different. Co-occurrences in $\mathcal{T}$-space have yet another flavor. Each of the transitions $S \to G$ and $G \to \mathcal{T}$ is entropically favored. We now look at the former, the latter being the standard theory of quantum measurement.

As a first example, consider the scalar distinction $[1, \tilde{1}]$, an element of S, which is mapped to the vector a, an element of G, and therewith encapsulates one bit, cf. Table 5. $[1, \tilde{1}] \in S$ because both 1 and $\tilde{1}$ must be simultaneously present if the idea of their distinction is to be meaningful. Thus, what is a *superposition* of 1 and $\tilde{1}$ in S becomes an *alternation* between 1 and $\tilde{1}$ in $a \in G$. A degree of freedom has been lost.

A second example: the co-exclusions $(a + \tilde{b} \,|\, \tilde{a} + b)|(a + b \,|\, \tilde{a} + \tilde{b})$ induce the formation of ab. What happens is that the *superpositions* in S represented by the co-exclusions - three of them - have been replaced by their actualized *alternations*, $(a+\tilde{b} \leftrightarrow \tilde{a}+b) \leftrightarrow (a+b \leftrightarrow \tilde{a}+\tilde{b})$ in G. [26] That is, the superpositions in S are replaced by space-like exclusions in G, which is, again, a *reduction* in the number of states. In the next step, this reversible alternation in G is replaced by *before/after*, that is, it becomes a time-like (irreversible) exclusion in $\mathcal{T}$.

The overall movement of information is thus from superposition in S to space-like exclusion ("alternation") in G to time-like exclusion ("before-after") via projection/measurement in $\mathcal{T}$. Each of these steps increases entropy by (further) compartmentalizing information, which reduces correlation, ie. increases noise, which is entropy.

The information that Shannon defined is namely *time-like,* and is exactly modelled by a binary decision tree descent from root to leaf. In contrast, what δ does is to *build* that tree from the leaves (detailed co-occurrences like $a+\tilde{b}$) first to ab, ie. $\delta(a+\tilde{b}) = ab$, and from there up to the root $abc\ldots z$. In doing so, it reduces the information content of S by turning its superpositions into exclusionary distinctions in G, which in turn, at level 4, are projected into $3+1$d tauquernion spacetime. The Bit Bang explosion is much like the irresistible salesman who argues that owning one cow *after* the other is really just as good as owning two cows at the *same* time. (Although it isn't, as we know.)

When we calculate the information content of $G = \Psi$, we are counting non-Shannon information. And yet, the conceptual *basis* for this counting up of non-Shannon information is Shannon's time-like information, information you can use to locate and identify things in a space, cf. the binary tree descent! This is the "subtle paradox" mentioned in the first sentence of this section.

We resolve the paradox by viewing the entropic expansion $\mathcal{G}_0 \to \mathcal{G}_1 \to \mathcal{G}_2 \to \mathcal{G}_3 \to \mathcal{G}_4$ as the conversion of the space-like information in S and G into time-like information in $\mathcal{T}$-space - $e$bits, mass, 3d space, gravity, entropy, and time. That is, causal *potential* is converted into causal *actuality,* and it is in this conversion that the Shannon encoding of non-Shannon information is rendered meaningful, as namely Shannon information.

The continuation of this entropically-favored process of increasing encapsulation

$$a \xrightarrow{\delta} ab \xrightarrow{\delta} abc \xrightarrow{\delta} abcd \xrightarrow{\delta} \ldots \xrightarrow{\delta} abc\ldots z$$

would seem to lead to the conclusion that black holes are to be described by pseudo-scalars of grade $4n$, where n is *very* large, and "4" because this is a gravitational phenomenon, and the algebra cycles semantically $mod\,4$ (and more subtly, $mod\,8$). We are namely looking at (ie. inside) the interior of a gigantic gravitationally resonantly bound particle with 2^{4n} dimensions. At this

[26] Note that the co-exclusion form sums to 0, and so holds no contradiction.

extremely high level of gravitational organization (read heavily entangled), everything is so intensely correlated with everything else that, in the limit, all entities become indistinguishable from each other. In this way, the stage is set for a new expansion. [27]

8. Cosmological Evolution

The preceding section dilineated the information content of elements of the algebra, and thereafter how these elements are stitched together computationally and mathematically (namely with co-exclusion $\mapsto \delta$) to create ever more actualized structures. Left unaddressed however, is how exactly these algebraic elements come to be in the first place.

Metaphysics aside, we rely on two pillars of support in this telling of this story:

- The structure of the algebra itself, without questioning whether this is putting too much in by hand.
- The entropic propensity, ie. the truth of the 2^{nd} Law of Thermodynamics.

These are the governing principles in what follows.

The overall story arc is that the information creation via co-occurrence (cf. the Coin Demo), which is both dominant and non-Shannon, can be sustained using reversible mechanisms. The result is an exponentially expanding space-like information space, namely $G = \Psi$. This information is then bled off by its conversion into its time-like form, which we experience as $\mathcal{H}, \mathcal{M}, \mathcal{D}$, the Big Bang, and its aftermath.

The primitive mechanisms that contribute to the creation of bits of information are

- Distinctions: scalar 1 vs. $\tilde{1}$, and (multi-)vector $XY = -YX$
- Products, XY
- Co-occurrences, $X + Y$

[27] We note that the Pythagorean relationship $B_1^2 + B_2^2 = (B_1 + B_2)^2$ for the total entropy of the merge of two black holes B_1, B_2 [21] is satisfied by any two tauquernions so long as they anti-commute. See also the discussion of $\mathbb{Z}_n$ arithmetics in Appendix I.

The last of these dominates the information content of both S and $G = \Psi$ because the number of co-occurrences grows hyper-combinatorially. The two distinctions are clearly proto-bits. "Products" get their own line because, if co-occurrence is the steam locomotive, then products - being the generators of novelty - are the coal car, without a constant supply of which, the train will grind to a halt. This is detailed below.

Since we are dealing chiefly with co-occurrences, all "information" is non-Shannon unless otherwise noted. We are dealing only with *extant* elements of S, that is, with the elements of G as so far constructed. Abusing combinatorial notation, we are generating the set $\{\binom{\{1,a,b,\ldots\}}{m}\}$ of all the possible forms in $G = \{1, a, b, \ldots\}$. This generates $\Sigma\binom{n}{m} - 1 = \underset{1}{\Sigma}\binom{n}{m} = 2^n - 1$ elements.

The reason that the formula is $\underset{1}{\Sigma}\binom{n}{m}$, ie. leaving out one possibility ($m = 0$), is that $\mathcal{V}oid$ cannot be a party to a co-occurrence. This is because by definition, 0 means "does not occur", in the sense that $\mathcal{V}oid$ does not "happen", does not "take place", in either space or time, as opposed to the mis-understanding "not there at all". Thinking back to the Coin Demonstration, it simply cannot be performed when there is $\mathcal{N}oThing$ in the man's hand, but this does not deny $\mathcal{V}oid$'s presence.

We begin our construction with the scalars, G_0. These are $\mathcal{V}oid \mapsto 0$ and the primitive distinction $[1, \tilde{1}]$ that emerges from $\mathcal{V}oid$ [12]. The scalars have no dimensionality but *can* represent a primitive distinction if one has two of them. Dimensionally, $\mathcal{V}oid \mapsto 0$ represents a point, and the two-valued distinction ± 1 is the prototype of a line.

Including $\mathcal{V}oid$, G_0 has three distinctions $[\neg\mathcal{V}oid, \neg 1, \neg\tilde{1}]$ leading to $lg\,3 = 1.58$ bits; counting just the two non-zero states, this represents $lg\,2 = 1.00$ bit. These two different bit-measures express the difference between the space S of *possibilities*, and the space G of *extant* (in Ψ) entities, ie. those that have actually been constructed out of the possibilities.

The transition from G_0 to G_1 maps the scalar distinction $[1, \tilde{1}]$ to a 1-vector, a. This is an entropically favorable transition, according to Table 5, because a has one bit less information than the scalars from which it is formed. This mapping reifies into an exclusion what previously was only a potential to be 1 or $\tilde{1}$. Both scalars and vectors are now present, and Table 5 shows that they always have the highest information content of all.

The forms $\underset{1}{\Sigma}\binom{2}{m}$ of G_1, which we might also write as $\underset{1}{\Sigma}\binom{\{1,a\}}{m}$, yield the set $\{1, a, 1+a\}$, but $\delta(1+a) = a$, which we already have, so no novelty is generated.

The expansion must therefore seek another route ... which is (to await) the co-occurrence $a + b$, wherein we imagine the parallel existence of many G_1's (this is all an idealization, of course). Once a and b co-occur, they can co-exclude, whence ab, *a new entity*, is added to G. This is the coal car feeding the steam engine: every time a new entity is added to G, the number of co-occurrences, the size of G, doubles. [28]

Note that even though the multiplication $a+b \mapsto ab$ is reversible (eg. $a(ab) = b$), information is nevertheless created when ab is created (2.17 vs. 1.17 bits). As noted in §7.1, what is going on is that bins - of *possibility* - are simply being visited.

Addition (co-occurrence) is doing most of the work of the expansion - it's always entropically favored. But multiplication supplies a vital piece, namely the step from $a + b$ to ab. This being a crucial step, we reason that ab has the same information content as a and b, so in multiplying the latter together, it's $1 \times 1 = 1$ so to speak: we are simply combining things of the same measure and nothing is being "manufactured". Nevertheless, ab is still *novel*, so in the context of S and G and their basis in co-occurrences, we still harvest an information windfall from ab's appearance, because this gives (entropically favored) birth to a whole new generation of co-occurrences.

This may sound dodgy - something for nothing is always suspect - but the mathematics speaks clearly. It is non-Shannon (ie. space-like) information that becomes available via (though not because of) space-like rotation, $G = \Psi$ is expanding (because of addition), and there is no time-like context here.

This reasoning applies to all co-occurrences and products, and thus the expansion of Ψ is a general free-for-all application of co-occurrence $+$ and action $\times$ over and between all extant entities, biased in the general direction of entropy generation. But we are ahead of the story, and now must back up.

Eventually, all the elements of our G_1, call it $G_1^a = \{1, a\}$, will have been generated, so we must await a co-occurrence with a new entity, call it $b \in G_1^b$, and we then can generate $G_1^a + G_1^b$. Recall that co-occurrences always have a lower information content than the singletons composing them, so $G_1^a + G_1^b$ is entropically favored.

Once there is co-occurrence, there can be action: G_2 is created by $G_1^a \times G_1^b =$ $\{1, a, \} \times \{1, b\} = \{1, a, b, ab\} = G_2^{ab}$. Besides $q$bits, this produces, in particular, the high-information bivector ab, and thence W/Z and neutrinos.

Nevertheless, at some point, the combinatorial possibilities of G_2^{ab} too will be realized, whence we await a co-occurrence with an entity belonging to another G, say $G_2^{ab} + G_1^c$, leading to the product $G_2^{ab} \times G_1^c$:

[28] Strictly speaking, we should not count 1-vectors and pseudo-vectors, the $\binom{n}{1}$ and $\binom{m}{m}$ terms of the Σ, since we're counting co-occurrences, and these are singletons. On the other hand, including $\mathcal{O}(n)$ singletons has negligible impact on $\mathcal{O}(2^n)$.

$$G_3^{abc} = \{1, a, b, ab\} \times \{1, c\} = \{1, a, b, c, ab, ac, bc, abc\}$$

With G_3^{abc} we get photons, electrons, quarks, protons, neutrons, mesons, gluons - all the familiar members of the Standard Model.

Similarly, $G_2^{ab} \times G_2^{cd}$ and $G_1^{d} \times G_3^{abc}$ together generate G_4^{abcd} - giving us $\mathcal{H}, \mathcal{M}, \mathcal{D}$, $3+1$ spacetime, mass, gravity, and entropy - at which point we leave quantum mechanics. $G_{4n} \times G_{4n}$ describe higher-order gravitational structures.

However, we have again gotten ahead of our story. In generating G_2 from $G_1 \times G_1$, we can further imagine the co-occurrence and subsequent product of several (say four) G_1's (over, say, a, b, c, d), which will then produce the six bivectors ab, ac, ad, bc, bd, cd.

Once again we recall that co-occurrences always have a lower information content than the singletons that compose them, so entities like $ab+cd$ will again be entropically favored. These are, of course, $\mathcal{T}$'s ($\Rightarrow$ Bell/Magic states and $e$bits), and so we see that there is an entropically favored route to $\mathcal{H}$ and $\mathcal{M}$. [The same applies to $xy+xz$ (electrons) and $x+yz$ (quarks).] Since, all else seeming equal, there are three times as many $\mathcal{M}$ states as $\mathcal{H}$ states, the tendency here will be for the formation of normal matter.

Similarly, $G_1 + G_3$ will produce co-occurrences like $w + xyz$, the atoms of dark matter, so $\mathcal{D}$ is also an entropically favored outcome. Note that with the exception of ${}^{16}\mathcal{D}_u$, dark matter will be formed preferentially to normal matter, cf. $5.53, 6.87, 5.53$ versus 15.9 in Table 5. [Appendix II continues this discussion of combinatorial expansion.]

In both cases, the expansion is hyperexponential, and, being prior to the actual formation of 3+1d spacetime via the $\mathcal{T}$'s , is also not limited by the speed of light. Thus this combinatorial expansion presumably models the initial inflationary episode of standard cosmology.

Summarizing the cosmological development, both graphs in Figure 1 show the two major pathways to space/mass creation: upward on the left, the creation of $3+1$d space and normal matter, $\delta(\mathcal{H} \cup \mathcal{M}) = abcd$, via the pathway $\delta(\delta(a+b) + \delta(c+d)) = abcd$; and upward on the right, dark matter, via the pathway $\delta\mathcal{D} = \delta(d + \delta(c + \delta(a+b))) = abcd$, but then also for the latter, a "back door" down to $\mathcal{H} \cup \mathcal{M}$ via $\mathcal{D}_q^2$, $\mathcal{D}_u^2$, and $abc\,\mathcal{D}$ (cf. §6).

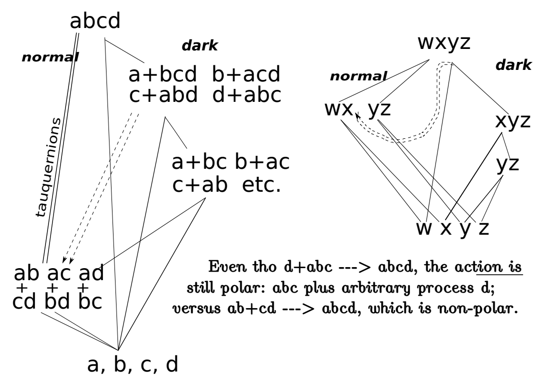

Figure 1: Two equivalent graphs of normal & dark matter creation. Growth (δ) is upward, as the ambient energy falls. The dotted lines symbolize the indirect tauquernion creation from dark-dark interactions.

9. Summary and Conclusions

We have a very promising candidate, the tauquernion $\mathcal{T}$- forms

$$\mathcal{H}, \mathcal{M} = \Sigma \binom{\{a,b,c,d\}}{2}, \quad \mathcal{H}^2 = 0, \quad \mathcal{M}^2 = abcd$$

for the long-sought connection between quantum mechanics and 3+1d relativity theory. This connection, which creates space, matter, and time, takes the form of a new, inherently entropic, way to describe 3d space. The conjugate forms of the Higgs bosons $\mathcal{H}$ presumably correspond to the dual polarizations of gravitational waves, and the members of $\mathcal{M}$ are the precursors of the unit mass *abcd*.

The overlap of $\mathcal{H} \cup \mathcal{M}$ and the entanglement states allows the partitioning of our understanding of matter and space into two complementary views: The tauquernion view focuses on the formation of matter, $3+1$d space, and gravity; whereas the the Bell/Magic view focuses on how the space and the matter all interconnect to form the whole. In hindsight, these two functionalities - the formation of structures and their interconnection - surely do lie best on the *very same* foundation - which turns out to be the largest *even* sub-algebra of $\mathcal{G}_4 = \{1, ab, ac, bc, ad, bd, cd, abcd\}$. But that's hindsight.

A near cousin $\mathcal{D}$ of $\mathcal{H} \cup \mathcal{M}$, the largest *odd* sub-algebra of $\mathcal{G}_4$,

$$\mathcal{D} = \Sigma \left({}^{\{a,b,c,d\}}_{1,3} \right), \mathcal{D}^2 \in \{0, xy + xz + yz, (w + x)(y + z)\}$$

offers a uniquely believable candidate for dark matter that also connects to $\mathcal{H} \cup \mathcal{M}$ via secondary $\mathcal{T}$-based connections. Our analysis predicts three types of dark structure, one nilpotent, one space-like (in that these square to quaternion triples), and one material (being 8^{th} roots of unity). This latter has two forms in the proportion 16 : 80, one (20%) heavy (15.9 bits) and one (80%) light (5.53 bits).

As this last sentence indicates, we have calculated the information content of every expression in $\mathcal{G}_0$, $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$ and $\mathcal{G}_4$. The classification system we developed to do this is based on the observation that an algebraic expression that picks out a single row of its "truth table" uses the most algebraic terms in order to provide this most discriminating specification. The sign-counts ($\#+$'s, $\#-$'s, $\#0$'s) associated with an expression, which counts are as well invariant over symbol substitutions, fit this observation exactly. However, because many quite different expressions in $\mathcal{G}_4$ have the same count-signature, giving misleadingly high bin populations, our final classification algorithm therefore uses *both* these counts *and* the number of (non-scalar) terms in the expression - a Euclidean length - to choose a bin. Thus our binning algorithm compactly represents both the state and the algebraic complexity of any expression.

We explicitly iterated through all sign variants of all expressions in $\mathcal{G}_1$ (2 bins), $\mathcal{G}_2$ (4 bins), $\mathcal{G}_3$ (14 bins) and $\mathcal{G}_4$ (86 bins for 43 million expressions) in order to calculate the exact bin populations for each such signature. These in turn yield the highest bit value for the least likely bins (eg. m-vectors and single-row specifiers) and the lowest bit value for the most likely bins (eg. large concurrent expressions).

The biggest surprise was that primitive concurrency (addition of vectors/m-vectors) is easily the primary mechanism for information creation. While multiplication's transformative power is, as we saw (§8), necessary to maintain a supply of novel entities, the hyper-combinatorial state expansion fostered by additive combination of said novelty vastly exceeds the latter's numbers. The potential information so created is ultimately released as energy according to the relation 1 bit = 4 $Planck\ areas$ $/ln\,2$.

As the state space expands from $\mathcal{G}_1$ thru $\mathcal{G}_4$, the bit value of an individual m-vector grows from 0.58 to 18.9 bits due to the explosion in the size of the state space. This Bit Bang represents *real* bits that are released as *real* energy, the energy that fuels the Big Bang when converted to the (statistically likely) Higgs and mass states at 7.08 bits. Thus, for example, two bivectors at 18.9 bits each, combined concurrently (eg. yielding a $\mathcal{T}$), yield a co-occurrence with an information content of 15.1. This (probably) entangled $\mathcal{T}$-state persists due to irreversibility, and therefore has increased likelihood of forming (with two others) an element of $\mathcal{H} \cup \mathcal{M}$, with an ensuing huge further entropy increase to 7.08 bits. Globally and locally, the expansion process is monotonic due both to the irreversibility of the entangled Bell/Magic states and the entropic expansion in general.

In the standard QM story, the quantum potential Ψ is the home of superpositions, and the transition from superposed to definite states lies at the heart of the quantum mechanical world. Since in the same standard story there is no *mechanism* – it being entirely statistical in content – no finer distinctions were needed. Having with our computational interpretation introduced the missing mechanism, we were able to see the distinction between what can imaginably be ($\mathcal{S}$, superpositions), versus what can *potentially* exist ($G = \Psi$, alternations), versus what actually *is*, $\mathcal{T} \cdot G \mapsto 3 + 1$d. The distinction between superposition and alternation in turn allowed the formulation of a coherent story of entropic transformation from $\mathcal{S}$ to G to our own $3 + 1$d spacetime. Our information content calculations, besides being *exact* - a welcome rarity - seem consistent with both observation and standard theory, and as well fill in many details of what happens before the Big Bang bangs.

It seems appropriate now to remind the reader of the *hierarchical* structure of the algebra, and what it might mean when extended beyond $\mathcal{G}_4$. This structure has its foundation in the fact that the algebra's atoms – $a, ab, abc, \ldots$ – whose successive squares are the $+--+$ sequence of powers of i, are also "pure frequencies", since they are the dimensions onto which Parseval's Fourier decomposition projects, and simultaneously they *also* are oscillating co-exclusionary computations. Thus, in a sense, the $\mathcal{G}_3$ particle tables in §6 and Appendix I and their exact fit to the Standard Model are inevitable. At the same time, these m-vectors grow (via δ, cf. "symmetry-breaking") with the size of m in encoded complexity, such that one can only think that the detailed construction of hydrogen, helium, etc. is within reach, with molecular bonding and molecules next. Huygens' principle of secondary sources is a guide in this endeavor.

We note that the discovery of the tauquernions lends strong support to background-independent theories [4,9,13]. The tauquernion foundation for $3 + 1$d - via both the Higgs mechanism and entanglement - means that cosmological theories need no longer feel forced to *assume* the prior existence of 3+1d, as does eg. string theory. Rather, the availability of the tauquernions should encourage the development of background-free theories, which are for the same reason more conceptually satisfying.

Finally, we note that the Coin Demonstration delivers a decidedly *non-computable* bit of information (in the Turing sense), and would therefore seem to constitute the non-computable element sought by Penrose [18] and others.

All in all, we are very impressed with the deep correspondence of known and/or physically meaningful computational algebraic structures to their calculated information content, and of both of these to the physical phenomena they are meant to model. Even the subtlest processes seem almost to have been anticipated. Together with the present computational interpretation, the power and elegance of the $\mathbb{Z}_3$ geometric algebra can simply not be denied.

- - -

The minimalism of our $\mathbb{Z}_3$ dialect of geometric algebra has effortlessly and incredibly parsimoniously exhibited, via the tauquernions $\mathcal{T}$, virtually all the desired and necessary structures, seamlessly interwoven, to plausibly connect quantum mechanics to $3+1$d space-time, both its creation and its content. As a dividend, we also get a detailed structural theory of dark matter. The complete overlap of the $\mathcal{T}$ and entanglement spaces, making entanglement the mechanism of gravity, is a wonderful surprise. The information-theoretical analysis supplies a both detailed and *exact* "null hypothesis" backdrop for experiments. Hopefully, the more detailed formulation of this picture in unchained $\mathbb{Z}$, and its mapping to the body of general relativity and $\mathbb{R}$, will be straightforward, but nevertheless definitely a matter for professional physicists.

In this connection, we think it entirely reasonable that physicists expect, and even require, that the algebraic and interpretive framework that we have introduced provide the actual mechanisms for the physical effects we observe. Call this *information mechanics*. After all, we have presented a computational theory, and *mechanism* – what *must <u>actually</u> happen* – is the soul of the computational metaphor.

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See www.tauquernions.org for other papers by the authors.

Appendix I

The Standard Model in $\mathbb{Z}_3\ \mathcal{G}_3$

The $\mathbb{Z}_3\ \mathcal{G}_3$ Standard Model presented in this Appendix is in support of the preceding text, which provides algebraic context and other necessary details not found here, ie. this Appendix is *not* self-contained.

Our knowledge of the $\mathbb{Z}_3\,\mathcal{G}_3$ algebra has a strong empirical flavor, born of the fact that it takes only about eight seconds to search the entirety of $\mathcal{G}_3$ ($6581 = 3^8$ elements, versus days to weeks with $\mathcal{G}_4$), so instead of isolating abstract groups and proving theorems about their properties and inter-relationships, we just calculate and display all the expressions of interest. We can assure the reader that this Appendix rests on a thorough census of the forms in $\mathcal{G}_3$.

To the reader who would see actual abstract group elements paired off with elements of the algebra in accordance with the well-tested tenets of quarkology, we must plead ignorance. Thus the finer details of particle types and interactions, which *all* work out very nicely, are the algebra's hand at work - we have not attended to such things, nor needed to. While the presentation in the following pages more or less exhausts our knowledge of the subject, given the precision with which the algebra nails all the categories, *and* their details, plus the isomorphism between $\mathcal{G}_3$ and the Pauli algebra, we trust that any discrepancies will turn out to be technical and non-contradictory.

In the classifications that follow, the general reasoning is:

- $\mathbb{Z}_3\ \mathcal{G}$ is an algebra of *distinctions*, and every singleton $xy, xyz, wxyz, \ldots$ expresses a logical *xnor*, the negative of *xor*. Either way, it's the same/different distinction that is effected, and being in $\mathbb{Z}_3 = \{0, 1, -1\}$ ensures a binary classification over ± 1 (since never $x = 0$). This means that the $\mathbb{Z}_3$ algebra implicitly classifies all of its elements as same/different in intricate, yet minimal, combination; eg. unitary elements possess much sameness. This is another way to view an expression's information content.
- Stable particles U, V must be unitary, $U^2 = V^2 = 1$, whence their projectors are the idempotents $-1 \pm U, -1 \pm V$, whence bosons are the nilpotents ω that satisfy $(-1 \pm U)(-1 \pm V) = (-1 \pm U)(\omega)(-1 \pm V)$, thus indicating a causal sequence. Nilpotents and idempotents correspond, respectively, to the wait() and signal() synchronization primitives.
- The other classifications then follow from inner consistency and the Standard Model itself.

Excluding 1-vectors, the only three unitary forms in $\mathcal{G}_3$ are $x+y+xy$, $xy+xz$, and $x+y+z+xy+xz$, and have been found to correspond, respectively, to neutrinos, electrons, and protons (neutrons $=$ xyz protons).

1. Neutrinos:

Name	*Form*	*Vector* ($\mathcal{G}_2$)	*Signature*	*Bits*
ν	$a+b+ab$	$[- - -0]$	$(0,1,3),3$	1.75
ν_μ	$a-b-ab$	$[- - 0-]$	"	"
ν_τ	$-a+b-ab$	$[-0 - -]$	"	"
$\Sigma\ =$	$a+b-ab$	$[0 + + +]$	"	"
$\bar{\nu}$	$-a-b-ab$	$[+ + +0]$	"	"
$\bar{\nu}_\mu$	$-a+b+ab$	$[+ + 0+]$	"	"
$\bar{\nu}_\tau$	$a-b+ab$	$[+0 + +]$	"	"
$\Sigma\ =$	$-a-b+ab$	$[0 - - -]$	"	"

Although there are $2^3 = 8$ sign variants here, versus the Standard Model's six neutrinos, it turns out that in each half of the table, the fourth can be expressed as the sum of the other three. Indeed, this provides a framework for the mutation of one neutrino type into another, cf. "the solar neutrino problem".

We tentatively identify the nilpotent W and Z bosons as being of the form $x+xy$ (our only 'tentatives'), and one can imagine the sum $(x-xy)+(y-xy) = x+y+xy$, a neutrino. The forms $\imath = \pm 1 + x + xy$, $\imath^{3,6} = 1$, are also relevant.

Electrons can be formed the same way: $e = xy + xz = (x+xy) + (\tilde{x} + xz)$.

2. Electrons:

Name	*Form*	*Vector* ($\mathcal{G}_3$)	*Signature*	*Bits*
e	$ab+ac$	$[-00++00-]$	$(2,2,4),2$	4.70
$\bar{e}$	$-ab-ac$	$[+00--00+]$	"	"
e^-	$ab-ac$	$[0-+00+-0]$	"	"
$\bar{e}^-$	$-ab+ac$	$[0+-00-+0]$	"	"
μ	$ab+bc$	$[-0+00+0-]$	"	"
$\bar{\mu}$	$-ab-bc$	$[+0-00-0+]$	"	"
μ^-	$ab-bc$	$[0-0++0-0]$	"	"
$\bar{\mu}^-$	$-ab+bc$	$[0+0--0+0]$	"	"
τ	$ac+bc$	$[-+0000+-]$	"	"
$\bar{\tau}$	$-ac-bc$	$[+-0000-+]$	"	"
τ^-	$ac-bc$	$[00-++-00]$	"	"
$\bar{\tau}^-$	$-ac+bc$	$[00+--+00]$	"	"

3. Photons: $\pm x \pm y \pm z$. There are four pairs of 2 states γ, γ', which we take to be polarizations. Note that the electron projector $-1+xy+xz$ factors as $x(\tilde{x}+y+z)$; and that $\gamma\gamma' = 1 \pm (xy+xz)$. Also, $-1+xy+xz = (xy+yz-xz)(xyz)(\tilde{x}+y+z)$.

4. Mesons, Gluons, and E/M.

Like electrons, mesons too can be constructed via a 2-sum of the nilpotent $x+xy$ form, and gluons with a 3-sum. The sums that are factorable are nilpotent, and those that are not are roots of unity. We note that quarks have the form $x+yz$, and so mesons can easily consist of two quarks via rearrangement, cf. the first two items below:

- Nilpotent mesons: $\{X \mid X = (x+xz) + (y+yz) = (x+y)(1+z)$ & $X^2 = 0\}$
  (24) $\qquad\qquad = (x + yz) + (y + xz)$
- Massive mesons: $\{X \mid X = (x - xz) + (y + yz)$ & $X^2 = \pm xyz\}$
  (24) $\qquad\qquad = (x + yz) + (y - xz)$
- Gluons (48): $\{\mathfrak{g} \mid \mathfrak{g} = x + y + z + xy + xz + yz$ & $\mathfrak{g}^2 = \pm xyz\}$
- Electro-magnetic field: $\{E \mid E = (x + y + z) \pm xyz(x + y + z)$ & $E^2 = 0\}$
  (16) $\qquad\qquad = (1 \pm xyz)(x + y + z)$

Note that $xyz(x+y+z) = xy+xz+yz$ is the 3-space quaternion triple associated with the photon $x + y + z$, while $\pm xyz$ is the charge carrier. The last two items have the same form, differing only via charge vs. nilpotence. All four are eigen forms of xyz.

5. Quarks

The quarks are the only case where the $\mathcal{G}_3$ algebra at first seems insufficient, in that while the $x + yz$ form correctly exhibits three families of 2×2, with spin $(\pm xy, \pm xz, \pm yz)$ and charge ($\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ on x, y, z), in doing so it seems to use up all of its information carrying capacity, and then some, and so be unable to express as well the three colors quarks also can have.

It is appropriate therefore to enquire how a single 1-vector like x might even be *said* to carry *both* $\pm\frac{1}{3}$ charge *and* a color designation, especially since it carries only one bit of information. The answer is that x itself carries only the $\pm$ distinction, one bit. The "$\frac{1}{3}$" is *our imputation* of x's contribution to a larger pattern, and indeed the $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ charge-addition business is clearly the space-like non-Shannon information contained in a 3-co-occurrence, cf. the Coin Demonstration, where the answer to the question "is there electro-magnetism" is answered when the third coin is revealed.

Similarly, "color" is *our* way of distinguishing x from y from z, which is meaningful only when >1 are present. Since quarks and their colors appear only when there are either two (mesons) or three (hadrons, gluons) quarks present, so then also are the requisite co-occurring x, y, z's present. So we conclude that it is permissable to associate with each of x, y, z both a charge and a color.

We can encode the "colors" red, green, blue (r, g, b) as

$$\begin{array}{ccccccc} r & g & b & r+g & r+b & g+b & r+g+b \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ a & b & c & a+b & a+c & b+c & a+b+c \end{array}$$

Thus both charge and color are emergent, co-occurrence-based, non-Shannon distinctions. The finishing touch is that a particle and its anti-particle must sum to zero, including both charge and color. We then get the following table of quarks: [29]

Name	U	D	$\bar{U}$	$\bar{D}$
Form	$a+bc$	$-a+bc$	$-a-bc$	$a-bc$
Charge	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{1}{3}$
Color	r	$\bar{r}$	$\bar{r}$	r

Name	C	S	$\bar{C}$	$\bar{S}$
Form	$b+ac$	$-b+ac$	$-b-ac$	$b-ac$
Charge	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{1}{3}$
Color	g	$\bar{g}$	$\bar{g}$	g

Name	T	B	$\bar{T}$	$\bar{B}$
Form	$c+ab$	$-c+ab$	$-c-ab$	$c-ab$
Charge	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{1}{3}$
Color	b	$\bar{b}$	$\bar{b}$	b

6. Hadrons; Protons and Neutrons

$\mathcal{G}_3$ contains exactly three compound unitary forms X such that $X^2 = 1$. These are $x + y + xy =$ neutrinos, $xy + xz =$ electrons, and now the largest of these, the 96 hadron forms $x + y + z + xy + xz$, which square to either $1 \pm xyz$ or $+1$, 48 of each. Each 48 divides into three groups of 16, depending on which of the three possibilities $xy + xz$ occurs. By inspection, in the $X^2 = +1$ half, there are three sub-families, made up from the three families of quarks. Of the 16 in one such, the $8 + 8$ are each two photon polarizations γ and γ', the 8 dividing as $4 + 4 = 2 \times 2\ +\ 2 \times 2$, these being the 'conjugate' forms $\gamma \pm (xy + xz)$ and $\gamma \pm (xy - xz)$, and $\gamma' \pm (xy + xz)$ and $\gamma' \pm (xy - xz)$.

[29] Except for U and D, the entries in these tables were not assigned with any particular knowledge of how they are to correspond to real particles.

We saw earlier how the mesons can be constructed from two $x + xy$'s, and in so doing deftly confine the quarks so formed to a minimum presence of two. The same construction can be applied to the hadrons, which are then the sum of three $x + xy$'s, rearranged to make three $x + yz$'s.

In particular, protons are UUD and neutrons UDD, that is, $p = 2U + D$ and $n = 2D + U$. Subtracting these, $p - n = p + \bar{n} = 2U + D \; - \; 2D - U = U + \bar{D}$, ie. "$U\bar{D}$", a quark and an anti-quark, ie. a meson. Clearly, $n - p =$ "$\bar{U}D$", which symmetry is appropriate for an exchange particle like a meson. And indeed, the quark model stipulates that mesons be (the sum of) a quark and an anti-quark.

Unfortunately, in our $\mathbb{Z}_3$ algebra, $2U = \bar{U}$, so "count to 2" *also* means the "anti" distinction, and thus we cannot express the UUD vs. UDD distinction as things stand. Fortunately, we can move to $\mathbb{Z}_5 = \{\tilde{2}, \tilde{1}, 0, 1, 2\}$ and still remain in $\mathcal{G}_3$. [30] Being now able to count to 2, the quark model is straightforward. Let $U = a + bc$ and $D = -a + bc$. Then, with $\mathbb{Z}_5$ arithmetic,

$$\begin{gathered} p = 2U + D = 2(a + bc) + (-a + bc) \\ n = U + 2D = (a + bc) + 2(-a + bc). \\ \text{whence} \\ p - n = (2a + 2bc - a + bc) - (a + bc - 2a + 2bc) \\ = a + 3bc - (-a + 3bc) = 2a = (a + bc) + (a - bc) \\ \\ = \; U + \bar{D} \quad = \text{“}U\bar{D}\text{”} \end{gathered}$$

just as required; and we note that our proton $p = UUD$ has charge $\frac{4}{3} - \frac{1}{3} = +1$ and our neutron $n = UDD$ has charge $\frac{2}{3} - \frac{2}{3} = 0$. [31]

The success of the shift from $\mathbb{Z}_3$ to $\mathbb{Z}_5$ to clarify the quark model encourages the thought of $\mathbb{Z}_7$ for $\mathcal{G}_4$. This would emphasize the $0 \, mod \, 4$ cycle, which expands into itself: in the hierarchy of these algebras, they all will be $\mathcal{G}_{0 \, mod \, 4}$ because, abusing notation, $\delta(\mathcal{G}_{0 \, mod 4} + \mathcal{G}_{0 \, mod \, 4}) = \mathcal{G}_{0 \, mod \, 4}$. We believe this to be a black hole structure in the limit.

But in the first instance this leads to $\mathcal{G}_8$, octonions, and the exceptional Lie group E_8, well-known to string theorists. Perhaps $\mathbb{Z}_{11} = \{\tilde{5}, \tilde{4}, \ldots, 0, \ldots, 5\}$ is the right lens for $\mathcal{G}_8$. [32] The primes $3, 5, 7, 11$ appear initially for their

[30] We defer the interesting foundational question raised here, and instead take the pragmatic view that while Nature knows what it's doing, we need help focusing, and the shift to $\mathbb{Z}_5$ keeps the focus sharp.

[31] It is an interesting exercise to examine how the $\mathbb{Z}_3$ encoding of p (which of course must be equivalent) compensates for its inability to count to two by adding in extra and/or intertwined distinctions. Thus with $p = a + b + c + ab + ac = a + (b + ac) + (c + ab)$, two (non-$U, D$) quarks appear, and the a-distinction is decisive.

[32] On the other hand, we are not fans of octonion multiplicative non-associativity [11].

symmetry around 0, but as well, their self-identifying property correlates with the idempotent forms $\pm 1 \pm x_1 x_2 \ldots x_m$ of the corresponding level m, which in turn are the similarly self-identifying computational primitives ***signal(event)*** [11].

Returning to the $\mathbb{Z}_3$ algebra, we note that the proton form is also the sum of a photon and an electron. Consider now, in idempotent form, an electron $e = -1+xy+xz = x(\tilde{x}+y+z) = x\gamma$, and a proton, $p = -1+(\tilde{x}+y+z)+(xy+xz)$, which factors as $(\tilde{x}+y+z)+x(\tilde{x}+y+z) = (1+x)(\tilde{x}+y+z)$. Then

$$ep = (-1+xy+xz) \times (-1+\tilde{x}+y+z+xy+xz) = -1+xy+xz = e$$

$$= x(\tilde{x}+y+z) \times (1+x)(\tilde{x}+y+z)$$

$$= (\tilde{x}+\tilde{y}+\tilde{z})x \times (1+x) \times (\tilde{x}+y+z)$$

$$= (\tilde{x}+\tilde{y}+\tilde{z}) \times (1+x) \times (\tilde{x}+y+z)$$

$$= (\tilde{x}+\tilde{y}+\tilde{z})(1+x) \times x(\tilde{x}+y+z)$$

$$= (\tilde{x}+\tilde{y}+\tilde{z})(1+x) \times (\tilde{x}+\tilde{y}+\tilde{z})x$$

$$= (-1+\tilde{x}+\tilde{y}+\tilde{z}+xy+xz) \times (-1+xy+xz)$$

$$= p'e \qquad\qquad \textit{and} \qquad\qquad pe = e'p = p$$

where we note that the phase of the photon in p has changed from $\tilde{x}+y+z$ to $\tilde{x}+\tilde{y}+\tilde{z}$ in p'. So, even though the state $ep = p'e = e$ is nominally fixed (since the idempotents are irreversible) and officially static – it's what *has* happened and no more has happened *yet* – we see [tracing the movement of x] that there is a natural, reversible, electro-magnetic oscillation, or if you like, an indeterminacy of state, in the electron-proton interaction that is consistent with our identification of the photon, electron and proton forms.

Finally, the reader should note that *summing*, using which we have here described the build-up of the Standard Model's structure, ie. *co-occurrence*, is the entropically favored pathway for combining terms. However, the actual expansion is much more complicated than merely summing $x+xy$'s as we have done for expository purposes, which is, rather, simply a limited application of a spectral basis.[33]

[33] The general existence of a spectral basis for $\mathcal{G}$ is an open question.

Appendix II

The Combinatorial Hierarchy

[Continuing from the end of §8:]

There is one last point we wish to make concerning the generation of G_{i+j} from $G_i \times G_j$. Let $A = \{1, a\}$, whence we are in G_1. A-space is $\pm 1 \pm a \Rightarrow 2^2 = \mathbf{4} = 2^{2^1}$ states. Now let $B = \{1, b\}$. Then

$$A \times B = \{1, a, b, ab\}$$

and the resulting space is of size $2^4 = \mathbf{16} = \mathbf{4}^2 = 2^{2^2}$. The next step is

$$\{1, a\}\{1, b\}\{1, c\} = \{1, a, b, c, ab, ac, bc, abc\}$$

which is of size $2^8 = \mathbf{256} = \mathbf{16}^2 = 2^{2^3}$. Next is $\{1, a\}\{1, b\}\{1, c\}\{1, d\} =$

$$\{1, a, b, c, d, ab, ac, bc, ad, bd, cd, abc, abd, acd, bcd, abcd\}$$

which is of size $2^{16} = \mathbf{256^2} = 65538 = 2^{2^4}$.

The sequence of space-sizes increases as the square, $4 \rightarrow 16 \rightarrow 256 \rightarrow 256^2$, because of course $2^{2^n} = 2^{2^{n-1}} 2^{2^{n-1}}$. At the same time, the number of elements in these spaces (a subset of S) is growing even faster, and these two sequences are related. Table 6 shows the generation process, and the intertwining of the two sequences is visible in the related powers of 2 that appear.

In the early ($\mathbb{Z}_2$) analysis [1] of this construction - the *Combinatorial Hierarchy* - it was understood in terms of the state vectors of one level being stacked to make square matrices, which matrices had to be capable of mapping the resulting next-level space onto itself. The intriguing aspect then is that while the matrix, being a stack of basis vectors, exists for $n = 1, 2, 3$, at $n = 4$ the number of co-occurrences explodes, and the $((256)^2)^2 = 2^{32}$ basis vectors are completely swamped by the 2^{127} co-occurrences they should map among. That is, 4 covers 3, 16 covers 7, and 256 covers 127, but then it's over. So the construction halts, or must begin anew, or, at least, something *new* has to happen, seemed to be the message back then.

Lvl = n = *generators* $\rightarrow$	G_1	G_2	G_3	G_4
# *terms*	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$
Full state contents G	$\pm 1 \pm a$	$\pm 1 \pm a \pm b \pm ab$	$\pm 1 \pm a \pm b \pm c \pm$ $\pm ab \pm ac \pm bc \pm abc$	$\pm 1 \pm a \pm b \pm c$ $\pm d \pm ab \pm ac \pm bc$ $\pm ad \pm bd \pm cd \pm abc$ $\pm abd \pm acd \pm bcd \pm abcd$
$\lvert G\rvert = 2^{2^n} = (2^{2^{n-1}})^2$	$2^{2^1} = 2^2 = 4$	$2^{2^2} = 4^2 = 16$	$2^{2^3} = 16^2 = 256$	$2^{2^4} = 256^2 = 65538$
Occurrences $S_G = \sum_1 \binom{k}{j}$	$\sum_1 \binom{2}{j} = 3$	$\sum_1 \binom{3}{j} = 7$	$\sum_1 \binom{7}{j} = 127$	$\sum_1 \binom{127}{j} = 2^{127} - 1 \approx 10^{38}$
	$\{1, a, 1+a\}$	$\{a, b, a+b \xrightarrow{\delta} ab,$ $a+ab, b+ab, a+b+ab\}$	$\{a, b, c, ab, ac, bc,$ $a+ab, a+ac, \ldots\}$	$\{a, b, c, d, ab, ac, ad, bc, bd,$ $cd, a+ab, a+ac, a+ad, \ldots\}$

Table 6: *The Combinatorial Hierarchy, CH* [1,2].

[Three brief comments: (1) S_G is that part of S that corresponds to G's alternations; (2) the bottom two rows of the table show only $+$ variants because the signature collapses all sign variants to the same bin; and (3) the base of the combinatorics, 2-ary distinctions, is the one that generates the most structure: 3- and 4-ary distinctions cut off sooner, and 5-ary doesn't even get off the ground [1].]

The present ($\mathbb{Z}_3$) perspective sees something *new*: the line that is crossed is the one that separates localizable effects from distributed ones, ie. weak, strong, and electromagnetic from EPR and gravity. Either way, the cut-off occurs with consistent and physically meaningful interpretations, and it seems clear that the two instances of the *CH* ($\mathbb{Z}_2$ and $\mathbb{Z}_3$) are both isomorphic and being imbued with the same physical import.

Finally, the observations that $3+7+127=137 \approx \frac{1}{\alpha}$, α being the fine structure constant, and that $3+7+127+2^{127} \approx 10^{38}$ roughly approximates the electromagnetism : gravity ratio, plus the above-described interpretation, led Bastin and Kilmister to refine this purely combinatorial approach to $\frac{1}{\alpha}$. Their most recent result [2] calculates this to 137.036011393. vs. the measured 137.035999710(96). We note that Bagdonaite *et alia.* report [4] that the proton-electron mass ratio has not varied in the past 7 billion years.

References

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