The Standard Model in $\mathbb{Z}_3\ \mathcal{G}_3$

by

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The $\mathbb{Z}_3\ \mathcal{G}_3$ Standard Model presented here lacks an appropriate introduction, especially regarding the interpretation of the algebra, which is normally to be found in the papers to which this writing is an appendix, which provide algebraic context and other necessary details not found here, ie. this 'appendix' is *not* self-contained.

Our knowledge of the $\mathbb{Z}_3\,\mathcal{G}_3$ algebra has a strong empirical flavor, born of the fact that it takes only about eight seconds to search the entirety of $\mathcal{G}_3$ ($6581 = 3^8$ elements, versus days to weeks with $\mathcal{G}_4$), so instead of isolating abstract groups and proving theorems about their properties and inter-relationships, we just calculate and display all the expressions of interest. We can assure the reader that this Appendix rests on a thorough census of the forms in $\mathcal{G}_3$.

To the reader who would see actual abstract group elements paired off with elements of the algebra in accordance with the well-tested tenets of quarkology, we must plead ignorance. Thus the finer details of particle types and interactions, which *all* work out very nicely, are the algebra's hand at work - we have not attended to such things, nor needed to. While the presentation in the following pages more or less exhausts our knowledge of the subject, given the precision with which the algebra nails all the categories, *and* their details, plus the isomorphism between $\mathcal{G}_3$ and the Pauli algebra, we trust that any discrepancies will turn out to be technical and non-contradictory.

In the classifications that follow, the general reasoning is:

- $\mathcal{G}$ is an algebra of *distinctions*, and every singleton $xy, xyz, wxyz, \ldots$ expresses a logical *xnor*, the negative of *xor*. Either way, it's the same/different distinction that is effected, and being in $\mathbb{Z}_3 = \{0, 1, -1\}$ ensures a binary classification over ± 1 (since never $x = 0$). This means that the $\mathbb{Z}_3$ algebra implicitly classifies all of its elements as same/different in intricate, yet minimal, combination; eg. unitary elements possess much sameness. This is another way to view an expression's information content.
- Stable particles U, V must be unitary, $U^2 = V^2 = 1$, whence their projectors are the idempotents $-1 \pm U, -1 \pm V$, whence bosons are the nilpotents ω that satisfy $(-1 \pm U)(-1 \pm V) = (-1 \pm U)(\omega)(-1 \pm V)$, thus indicating a causal sequence. Nilpotents and idempotents correspond, respectively, to the wait() and signal() synchronization primitives.

- The other classifications then follow from inner consistency and the Standard Model itself.

Excluding 1-vectors, the only three unitary forms in $\mathcal{G}_3$ are $x+y+xy$, $xy+xz$, and $x+y+z+xy+xz$, and have been found to correspond, respectively, to neutrinos, electrons, and protons (neutrons $= xyz$ protons).

1. Neutrinos:

Name	*Form*	*Vector*	*Signature*	*Bits*
ν	$a+b+ab$	$[- - -0]$	$(0,1,3),3$	1.75
ν_μ	$a-b-ab$	$[- - 0-]$	"	"
ν_τ	$-a+b-ab$	$[-0 - -]$	"	"
$\Sigma\ =$	$a+b-ab$	$[0+ ++]$	"	"
$\bar{\nu}$	$-a-b-ab$	$[+ + +0]$	"	"
$\bar{\nu}_\mu$	$-a+b+ab$	$[+ + 0+]$	"	"
$\bar{\nu}_\tau$	$a-b+ab$	$[+0 + +]$	"	"
$\Sigma\ =$	$-a-b+ab$	$[0- - -]$	"	"

Although there are $2^3 = 8$ sign variants here, versus the Standard Model's six neutrinos, it turns out that in each half of the table, the fourth can be expressed as the sum of the other three. Indeed, this provides a framework for the mutation of one neutrino type into another, cf. "the solar neutrino problem".

We tentatively identify the nilpotent W and Z bosons as being of the form $x+xy$ (our only 'tentatives'), and one can imagine the sum $(x-xy)+(y-xy) = x+y+xy$, a neutrino. The forms $\imath = \pm 1 + x + xy$, $\imath^3 = 1$, are also relevant.

Electrons can be formed the same way: $e = xy + xz = (x+xy) + (\tilde{x} + xz)$.

2. Electrons:

Name	*Form*	*Vector*	*Signature*	*Bits*
e	$ab+ac$	$[-00++00-]$	$(2,2,4),2$	4.70
$\bar{e}$	$-ab-ac$	$[+00--00+]$	"	"
e^-	$ab-ac$	$[0-+00+-0]$	"	"
$\bar{e}^-$	$-ab+ac$	$[0+-00-+0]$	"	"
μ	$ab+bc$	$[-0+00+0-]$	"	"
$\bar{\mu}$	$-ab-bc$	$[+0-00-0+]$	"	"
μ^-	$ab-bc$	$[0-0++0-0]$	"	"
$\bar{\mu}^-$	$-ab+bc$	$[0+0--0+0]$	"	"
τ	$ac+bc$	$[-+0000+-]$	"	"
$\bar{\tau}$	$-ac-bc$	$[+-0000-+]$	"	"
τ^-	$ac-bc$	$[00-++-00]$	"	"
$\bar{\tau}^-$	$-ac+bc$	$[00+--+00]$	"	"

3. Photons: $\pm x \pm y \pm z$. There are four pairs of 2 states γ, γ', which we take to be polarizations. Note that the electron projector $-1+xy+xz$ factors as $x(\tilde{x}+y+z)$; and that $\gamma\gamma' = 1\pm(xy+xz)$. Also, $-1+xy+xz = (xy+yz-xz)(xyz)(\tilde{x}+y+z)$.

4. Mesons, Gluons, and E/M.

Like electrons, mesons too can be constructed via a 2-sum of the nilpotent $x+xy$ form, and gluons with a 3-sum. The sums that are factorable are nilpotent, and those that are not are roots of unity. We note that quarks have the form $x+yz$, and so mesons can easily consist of two quarks via rearrangement, cf. the first two items below:

- Nilpotent mesons: $\{X \mid X = (x+xz) + (y+yz) = (x+y)(1+z) \ \& \ X^2 = 0\}$
  (24) $\qquad = (x+yz) + (y+xz)$
- Massive mesons: $\{X \mid X = (x-xz) + (y+yz) \ \& \ X^2 = \pm xyz\}$
  (24) $\qquad = (x+yz) + (y-xz)$
- Gluons (48): $\{\mathfrak{g} \mid \mathfrak{g} = x+y+z+xy+xz+yz \ \& \ \mathfrak{g}^2 = \pm xyz\}$
- Electro-magnetic field: $\{E \mid E = (x+y+z) \pm xyz(x+y+z) \ \& \ E^2 = 0\}$
  (16) $\qquad = (1 \pm xyz)(x+y+z)$

Note that $xyz(x+y+z) = xy+xz+yz$ is the 3-space quaternion triple associated with the photon $x+y+z$, while $\pm xyz$ is the charge carrier. The last two items have the same form, differing only via charge vs. nilpotence. All four are eigen forms of xyz.

5. Quarks

The quarks are the only case where the $\mathcal{G}_3$ algebra at first seems insufficient, in that while the $x+yz$ form correctly exhibits three families of 2×2, with spin $(\pm xy, \pm xz, \pm yz)$ and charge ($\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ on x, y, z), in doing so it seems to use up all of its information carrying capacity, and then some, and so be unable to express as well the three colors quarks also can have.

It is appropriate therefore to enquire how a single 1-vector like x might even be *said* to carry *both* $\pm\frac{1}{3}$ charge *and* a color designation, especially since it carries only one bit of information. The answer is that x itself carries only the $\pm$ distinction, one bit. The "$\frac{1}{3}$" is *our* imputation of x's contribution to a larger pattern, and indeed the $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$ charge-addition business is clearly the space-like non-Shannon information contained in a 3-co-occurrence, cf. the Coin Demonstration, where the answer to the question "is there electro-magnetism" is answered when the third coin is revealed.

Similarly, "color" is *our* way of distinguishing x from y from z, which is meaningful only when >1 are present. Since quarks and their colors appear only when there are either two (mesons) or three (hadrons, gluons) quarks present, so then also are the requisite co-occurring x, y, z's present. So we conclude that it is permissable to associate with each of x, y, z both a charge and a color.

We can encode the "colors" red, green, blue (r, g, b) as

$$\begin{array}{ccccccc} r & g & b & r+g & r+b & g+b & r+g+b \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ a & b & c & a+b & a+c & b+c & a+b+c \end{array}$$

Thus both charge and color are emergent, co-occurrence-based, non-Shannon distinctions. The finishing touch is that a particle and its anti-particle must sum to zero, including both charge and color. We then get the following table of quarks: [1]

Name	U	D	$\bar{U}$	$\bar{D}$
Form	$a+bc$	$-a+bc$	$-a-bc$	$a-bc$
Charge	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{1}{3}$
Color	r	$\bar{r}$	$\bar{r}$	r

Name	T	S	$\bar{T}$	$\bar{S}$
Form	$b+ac$	$-b+ac$	$-b-ac$	$b-ac$
Charge	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{1}{3}$
Color	g	$\bar{g}$	$\bar{g}$	g

Name	C	B	$\bar{C}$	$\bar{B}$
Form	$c+ab$	$-c+ab$	$-c-ab$	$c-ab$
Charge	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$+\frac{1}{3}$
Color	b	$\bar{b}$	$\bar{b}$	b

6. Hadrons; Protons and Neutrons

$\mathcal{G}_3$ contains exactly three compound unitary forms X such that $X^2 = 1$. These are $x + y + xy =$ neutrinos, $xy + xz =$ electrons, and now the largest of these,

[1] Except for U and D, the entries in these tables were not assigned with any particular knowledge of how they are to correspond to real particles.

the 96 hadron forms $x+y+z+xy+xz$, which square to either $1 \pm xyz$ or $+1$, 48 of each. Each 48 divides into three groups of 16, depending on which of the three possibilities $xy+xz$ occurs. By inspection, in the $X^2 = +1$ half, there are three sub-families, made up from the three families of quarks. Of the 16 in one such, the $8+8$ are each two photon polarizations γ and γ', the 8 dividing as $4+4 = 2\times 2 \,+\, 2\times 2$, these being the 'conjugate' forms $\gamma \pm (xy+xz)$ and $\gamma \pm (xy-xz)$, and $\gamma' \pm (xy+xz)$ and $\gamma' \pm (xy-xz)$.

We saw earlier how the mesons can be constructed from two $x+xy$'s, and in so doing deftly confine the quarks so formed to a minimum presence of two. The same construction can be applied to the hadrons, which are then the sum of three $x+xy$'s, rearranged to make three $x+yz$'s.

In particular, protons are UUD and neutrons UDD, that is, $p = 2U+D$ and $n = 2D+U$. Subtracting these, $p-n = p+\bar{n} = 2U+D \;-\; 2D-U = U+\bar{D}$, ie. "$U\bar{D}$", a quark and an anti-quark, ie. a meson. Clearly, $n-p =$ "$\bar{U}D$", which symmetry is appropriate for an exchange particle like a meson. And indeed, the quark model stipulates that mesons be (the sum of) a quark and an anti-quark.

Unfortunately, in our $\mathbb{Z}_3$ algebra, $2U = \bar{U}$, so "count to 2" *also* means the "anti" distinction, and thus we cannot express the UUD vs. UDD distinction as things stand. Fortunately, we can move to $\mathbb{Z}_5 = \{\tilde{2}, \tilde{1}, 0, 1, 2\}$ and still remain in $\mathcal{G}_3$. [2] Being now able to count to 2, the quark model is straightforward. Let $U = a+bc$ and $D = -a+bc$. Then, with $\mathbb{Z}_5$ arithmetic,

$$\begin{gathered}
p = 2U+D = 2(a+bc)+(-a+bc) \\
n = U+2D = (a+bc)+2(-a+bc). \\
\text{whence} \\
p-n = (2a+2bc-a+bc)-(a+bc-2a+2bc) \\
= a+3bc-(-a+3bc) = 2a = (a+bc)+(a-bc) \\[2em]
= \;U+\bar{D}\; \;=\; \text{“}U\bar{D}\text{”}
\end{gathered}$$

just as required; and we note that our proton $p = UUD$ has charge $\frac{4}{3} - \frac{1}{3} = +1$ and our neutron $n = UDD$ has charge $\frac{2}{3} - \frac{2}{3} = 0$. [3]

The success of the shift from $\mathbb{Z}_3$ to $\mathbb{Z}_5$ to clarify the quark model encourages the thought of $\mathbb{Z}_7$ for $\mathcal{G}_4$. This would emphasize the $0\,mod\,4$ cycle, which expands

[2] We defer the interesting foundational question raised here, and instead take the pragmatic view that while Nature knows what it's doing, we need help focusing, and the shift to $\mathbb{Z}_5$ keeps the focus sharp.

[3] It is an interesting exercise to examine how the $\mathbb{Z}_3$ encoding of p (which of course must be equivalent) compensates for its inability to count to two by adding in extra and/or intertwined distinctions. Thus with $p = a+b+c+ab+ac = a+(b+ac)+(c+ab)$, two (non-$U, D$) quarks appear, and the a-distinction is decisive.

into itself: in the hierarchy of these algebras, they all will be $\mathcal{G}_{0\,mod\,4}$ because, abusing notation, $\delta(\mathcal{G}_{0\,mod4} + \mathcal{G}_{0\,mod\,4}) = \mathcal{G}_{0\,mod\,4}$. We believe this to be a black hole structure in the limit.

But in the first instance this leads to $\mathcal{G}_8$, octonions, and the exceptional Lie group E_8, well-known to string theorists. Perhaps $\mathbb{Z}_{11} = \{\tilde{5}, \tilde{4}, \ldots, 0, \ldots, 5\}$ is the right lens for $\mathcal{G}_8$. [4] The primes $3, 5, 7, 11$ appear initially for their symmetry around 0, but as well, their self-identifying property correlates with the idempotent forms $\pm 1 \pm x_1 x_2 \ldots x_m$ of the corresponding level m, which in turn are the similarly self-identifying computational primitives ***signal(event)*** [8].

Returning to the $\mathbb{Z}_3$ algebra, we note that the proton form is also the sum of a photon and an electron. Consider now, in idempotent form, an electron $e = -1+xy+xz = x(\tilde{x}+y+z) = x\gamma$, and a proton, $p = -1+(\tilde{x}+y+z)+(xy+xz)$, which factors as $(\tilde{x}+y+z) + x(\tilde{x}+y+z) = (1+x)(\tilde{x}+y+z)$. Then

$$ep = (-1 + xy + xz) \times (-1 + \tilde{x} + y + z + xy + xz) = -1 + xy + xz = e$$

$$= x(\tilde{x} + y + z) \times (1 + x)(\tilde{x} + y + z)$$

$$= (\tilde{x} + \tilde{y} + \tilde{z})x \times (1 + x) \times (\tilde{x} + y + z)$$

$$= (\tilde{x} + \tilde{y} + \tilde{z}) \times (1 + x) \times (\tilde{x} + y + z)$$

$$= (\tilde{x} + \tilde{y} + \tilde{z})(1 + x) \times x(\tilde{x} + y + z)$$

$$= (\tilde{x} + \tilde{y} + \tilde{z})(1 + x) \times (\tilde{x} + \tilde{y} + \tilde{z})x$$

$$= (-1 + \tilde{x} + \tilde{y} + \tilde{z} + xy + xz) \times (-1 + xy + xz)$$

$$= p'e \qquad \textit{and} \qquad pe = e'p = p$$

[4] On the other hand, we are not fans of octonion multiplicative non-associativity [8].

where we note that the phase of the photon in p has changed from $\tilde{x}+y+z$ to $\tilde{x}+\tilde{y}+\tilde{z}$ in p'. So, even though the state $ep = p'e = e$ is nominally fixed (since the idempotents are irreversible) and officially static – it's what *has* happened and no more has happened *yet* – we see [tracing the movement of x] that there is a natural, reversible, electro-magnetic oscillation, or if you like, an indeterminacy of state, in the electron-proton interaction that is consistent with our identification of the photon, electron and proton forms.

Finally, the reader should note that *summing*, using which we have here described the build-up of the Standard Model's structure, ie. *co-occurrence*, is the entropically favored pathway for combining terms. However, the actual expansion is much more complicated than merely summing $x + xy$'s as we have done for expository purposes, which is, rather, simply an application of a spectral basis.

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